Money and Credit Redux*

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Abstract

Are money and credit both essential? We show in a standard model the answer is – *no*. If credit is easy, money has no role. If credit is tight, money can be valued, but then changes in debt limits are *neutral*. This is true for any (not only optimal) monetary policy, and whether debt and tax limits are exogenous or endogenous; it can be overturned by introducing certain (not all) types of heterogeneity or collateralized borrowing. Those who think credit matters should check whether their theories survive the introduction of currency, and whether they require specific types of collateral or heterogeneity.

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"Of all branches of economic science, that part which relates to money and credit has probably the longest history and the most extensive literature." Lionel Robbins, *Introduction*, in von Mises (1953).

1 Introduction

An genuinely classic issue in economics concerns the relationship between money and credit, alternative instruments for the facilitation of intertemporal exchange. We take up the issue by asking a simple question: One sees money and credit both used in everyday practice, but do we *need* both? It is important to have theories with both for many reasons (e.g., to analyze monetary policy), but it is not so easy. Clearly, one needs to deviate from the frictionless paradigm of Debreu (1959), where all exchange is organized in one grand market in terms of date- and state-contingent commodities. One can decentralize equilibrium allocations in that model using alternative market structures, including spot markets plus one-step-ahead Arrow (1964) securities. These securities are contingent claims to numeraire next period and hence constitute a form of credit – issuing one gives you purchasing power now with an obligation due later. But because we can accomplish neither more nor less with securities than contingent commodity trade, we say this form of credit is *not essential*.[1]

If we have contingent commodity markets – and there is no reason in Debreu's world not to – it is uninteresting to study credit using classical general equilibrium theory. It is even less interesting to study money. Our alternative approach uses a standard New Monetarist model, as described in recent surveys by Williamson and Wright (2010*a*,*b*) or Nosal and Rocheteau (2011). This approach begins

[1] In case it is not apparent, we use the term essential in a technical sense (see Wallace 2010, who attributes the notion to Frank Hahn). Institution I (e.g., money or credit) is essential if the set of incentive-feasible allocations with I contains points that are not in the feasible set without I. See also Townsend (1987, 1988), Kocherlakota (1998), Wallace (2001), Mills (2007), Araujo and Minetti (2011), Araujo and Carmago (2012) and Gu et al. (2012).

with the premise that agents trade with each other, and not merely against their budget lines. This requires articulating an explicit environment, including preferences and technologies, as well as frictions like spatial or temporal separation and information or commitment problems. Then we can start to ask *how* agents trade – e.g., do they use barter, money or credit? And we can consider options for determining the terms of trade, including price taking, bargaining and more abstract mechanisms, in addition to competitive price taking.[2]

Following this method, it is not trivial to get money and credit coexisting, as assumptions often adopted to make one viable can make the other untenable. As a leading example, Kocherlakota (1998) formalizes the idea that money is a substitute for credit: if perfect credit is feasible, there is no need for money; if there is a lack of commitment and information (monitoring or record keeping), money can have a role, but these frictions preclude credit. Models like Kiyotaki and Wright (1989, 1993), e.g., have lack of commitment and information and hence an essential role for money, but have little to say about credit, with exceptions noted below. And models along the lines of Kehoe and Levine (1993) or Alvarez and Jermann (2000) provide a nice foundation for endogenous credit conditions, but usually ignore money, again with exceptions such as Aiyagari and Williamson (1999). One can view this project in part as contributing to a continuing endeavor to integrate microfounded models of money and credit.

In terms of other related work, in Townsend (1989), private information, spa-

[2]Further on the methodogloical front, for the issues at hand, one ought not simply *assume* exogenous credit restrictions, missing markets, incomplete contracts etc. – although something like that may *emerge* as an outcome. As Townsend (1988) says: "theory should explain why markets sometimes exist and sometimes do not, so that economic organization falls out in the solution to the mechanism design problem." As regards money and credit, in particular, Townsend (1989) asks "Can we find a physical environment in which currency-like objects play an essential role in implementing efficient allocations? Would these objects coexist with ... credit?" To think seriously about these questions it is an obvious nonstarter to impose at the outset an exogenous partition of commodity space into cash goods and credit goods, as in Lucas and Stokey (1987).

tial separation and limited communication imply currency can circulate among strangers while credit is used among those who know each other (see also Corbae and Ritter 2003 or Jin and Temzelides 2004). In Cavalcanti and Wallace (1999a,b) some agents can be monitored while others cannot, and in Kocherlakota and Wallace (1998) all agents can be monitored but only with a lag. Imperfect monitoring is also part of our story, although we model it differently, by assuming deviant behavior (reneging on obligations) is only detected probabilistically.[3] Telyukova and Wright (2008) adopt a different approach, getting credit into the monetary model of Lagos and Wright (2005) by having a round of decentralized trade alternating with multiple rounds of centralized trade, but that kind of credit is no more essential than Arrow's securities in Debreu's economy. Still, we share with those papers the structure of alternating centralized and decentralized trade.

Another approach due to Shi (1996) studies models where money and credit are complementary: consumers can get credit, but later needs to settle in cash. Related, in some models consumers need cash, but to get it they may need loans (see Berentsen et al. 2007, Ferraris 2010, Ferraris and Watanabe 2008, Li and Li 2011, Geromichalos and Herrenbrueck 2011 or He et al. 2012). By contrast, money and credit here are not complements, but substitutes, as alternative ways to facilitate intertemporal trade. This is similar to Sanches and Williamson (2010). A problem in this kind of setting, however, is that money in a sense works too well: we can achieve efficiency with money and without credit by implementing a simple deflationary policy. One response is to introduce some unfavorable property of money, and following He et al. (2005, 2008), Sanches and Williamson assume cash is subject to theft while credit is not – which is fine, even if one can

[3]Imperfect monitoring is is important for reasons discussed in Wallace (2013): "If we want both monetary trade and credit in the same model, we need something between perfect monitoring and no monitoring. As in other areas of economics ... extreme versions are both easy to describe and easy to analyze. The challenge is to specify and analyze intermediate situations."

argue that using credit is actually riskier on this dimension in an age of identity theft (Khan et al. 2005).

In any case, we want to know if credit and money can both be essential without ad hoc assumptions that make either one somehow intrinsically bad. Hence, we follow Andolfatto (2007), who derives endogenous constraints on fiscal and monetary policy, similar to those on borrowing, that may make deflation infeasible beyond a threshold. Thus, limits on policy emerge because agents can renege on public obligations (taxes), just like they can renege on private obligations (debts). Combining these ingredients, to summarize the paper, we ask if it is useful to have both money and credit when they are substitutes as instruments for intertemporal exchange, and where limits to debt and deflation arise endogenously. Further, we study this in economies with some centralized and some decentralized exchange, limited commitment, stochastic monitoring, and a general class of mechanisms for determining the terms of trade.

By way of preview, here are some results: (1) In a benchmark model, with exogenous policy and debt limits, we show this: when debt limits do not bind, money cannot be valued and the credit-only outcome is efficient; if debt limits bind, money can be valued if and only if inflation is not too high. (2) While binding debt limits imply there are equilibria where cash and credit are both used, the latter is not essential, and changes in credit conditions are neutral (i.e., allocations are independent of the debt limit). Heuristically, if we start in a cash-only economy, then introduce some credit, real balances are *crowded out* one-for-one with no impact on total liquidity. If we allow enough credit, we can do better than the cash-only allocation, but only once cash is completely crowded out. Within the bounds consistent with the existence of monetary equilibrium, changes in debt limits are *neutral*.

Next we show: (3) The results on essentiality and neutrality hold when the framework is extended to endogenous policy and debt limits. (4) They also hold for any mechanism for determining the terms of trade that satisfies some natural axioms, including creatively-designed mechanisms, like the one in Hu et al. (2009), that sometimes allow us to achieve desirable outcomes even with inflationary policy. (5) The results hold when the model is extended to allow quite general types of shocks that lead to heterogeneity in transactions, but not in buyers' money holdings; the results may not hold when shocks lead to heterogeneity in buyers' money balances, or if there is heterogeneity in their debt limits. (6) The results hold when instead of unsecured lending we use debt collateralized by assets in fixed supply, like equity in Lucas (1978) trees, or land, as in Kiyotaki and Moore (1997); they may not hold when credit is collateralized by reproducible assets that enter production or utility functions, like capital or housing.

In points (5) and (6) above, we both argue for the robustness of the results and provide counterexamples, the idea being to uncover just what we need for the results to hold and to fail. The motivation for introducing shocks making some trades bigger than others comes from the old idea that agents often use cash for small and credit for big payments (Prescott 1987; Freeman and Huffman 1991). Having shocks ex post, after agents choose money balances, generates heterogeneity in purchases without overturning the main results. Having shocks ex ante, before they choose money balances, can break the results, as can heterogeneity in debt limits. Heuristically, point (5) says that unsecured credit can be essential and changes in credit conditions can be non-neutral when they involve redistribution across agents. And point (6) says that with secured lending credit conditions can have effects due to something like a Mundell-Tobin effect, as described below.

One reason all this is relevant is the following: Many economists seem to believe that credit conditions impinge on the economy in a big way – see, e.g., the survey by Gertler and Kiyotaki (2010), recent work by Williamson (2012), and the many primary references therein. Our results constitute some words of caution. In the baseline model, with unsecured lending and (roughly speaking) homogeneous buyers, as long as the economy is monetary, changes in credit conditions simply crowd out real balances with no net impact on total liquidity, as defined below. Given this, perhaps those economists ought to check whether their theories survive the introduction of currency. We also provided ways to break the results with certain types of shocks, depending on the timing, and with secured lending, depending on the nature of collateral. Again, those who think credit conditions matter perhaps ought to check if their models have the right ingredients – i.e., the right types of shocks, heterogeneity, or collateral – once currency is introduced.

The rest of the paper is organized as follows. Section 2 presents the basic environment. Section 3 discusses equilibrium with exogenous policy and debt limits, as well as a certain class of pricing mechanisms. Section 4 justifies the class of mechanisms. Section 5 endogenizes limits on debt and deflation. Section 6 considers extensions. Section 7 concludes.

2 Environment

Time is discrete and continues forever. In each period two markets convene sequentially: first there is a decentralized market, or DM, with frictions as detailed below; second there is a frictionless centralized market, or CM. Each period in the CM, a large number of infinitely-lived households work, consume, adjust their portfolios and settle their debt/tax obligations – or renege on these obligations, as the case may be. In the DM, some households called *sellers*, denoted by s, can produce but do not want to consume, while others called *buyers*, denoted by b, want to consume but cannot produce. Buyers and sellers in the DM trade bilaterally in the baseline model, but we discuss where appropriate how one can also have them trade multilaterally. So, for now, they meet pairwise and at random in the DM, with α denoting the probability that a buyer meets a seller.[4]

The period utility functions of buyers and sellers are $\mathcal{U}^b(q, x, \ell)$ and $\mathcal{U}^s(q, x, \ell)$, where q is a good consumed in the DM, x is a different good consumed in the CM, and ℓ is leisure. Labor is $1-\ell$, and there is a technology that converts a unit of labor into w units of the CM good x, so w is the equilibrium real wage (the results easily generalize to nonlinear technologies). There are constraints $x \geq 0$, $q \geq 0$ and $\ell \in \left[0, \bar{\ell}\right]$, but they are assumed not to bind, which can be guaranteed in the usual ways. To simplify the algebra let

$$\mathcal{U}^b(q, x, y) = u(q) + U^b(x, \ell) \text{ and } \mathcal{U}^s(q, x, y) = -v\left(q\right) + U^s(x, \ell), \qquad (1)$$

with U^j, u and v twice continuously differentiable and strictly increasing. Also, $u'' \leq 0 \leq v''$ with one equality strict, U is concave and $u(0) = v(0) = 0$.

Further, we adopt the following restriction from Wong (2012):

Assumption 1 $|U^j| = 0$, *where* $|U| = U_{11}U_{22} - U_{12}^2$.

This is satisfied for any quasi-linear utility function, $U = \tilde{U}\left(x\right) + \ell$ or $U = x + \tilde{U}\left(\ell\right)$, but also for a much broader class, including any U that is homogeneous

[4] To endogenize α we could specify a general matching technology, with or without entry on one side of the market, as in Pissarides (2000). The results all go through. Also, instead of households trading with each other in the DM, they can trade with producers or retailers, without changing the results. In fact, almost all of the interesting action concerns buyers, with sellers playing a minor role. Also, we do not actually need search: instead of saying buyers in the DM have an arrival rate of trading partners α, we can alternatively say α is the probability of a preference shock, and any buyer hit with the shock can immediately find a seller. In either case, our DM captures what Keynes (1936) called a *precautionary demand* for liquidity, defined in terms of providing "for contingencies requiring sudden expenditure and for unforeseen opportunities of advantage."

of degree 1, like $U = x^a \ell^{1-a}$ or $U = (x^a + \ell^a)^{1/a}$. The usefulness of Assumption 1 should be clear below.

Between the CM and the next DM, there is a discount factor $\beta = 1/(1+r)$, where $r > 0$ is the rate of time preference. There is no explicit discounting between the DM and CM, without loss of generality, because we can always renormalize CM utility by multiplying it by any discount factor. Goods q and x are perfectly divisible and nonstorable. There is also an intrinsically worthless object, called money, that is divisible and storable. The supply M changes over time at rate μ, so that $M_{+1} = (1+\mu) M$, where the subscript on M_{+1}, or any other variable, indicates its value next period. Changes in M are accomplished by lump sum transfers if $\mu > 0$ or taxes if $\mu < 0$. We restrict attention to $\mu > \beta - 1$, although we also consider the limiting case where $\mu \to \beta - 1$, which is the Friedman rule; there is no monetary equilibrium with $\mu < \beta - 1$.

There are two standard ways to model money or credit. One is to assume a desire for smooth consumption in the presence of fluctuating resources. Another is to assume a desire to satisfy random consumption needs or opportunities. It is convenient to use the latter, but it is not critical – all we really need is an imperfect synchronization of households' resources and consumption. Thus, in the DM, with probability α buyers have opportunities to trade with sellers and get q. For payments, in principle, they can use cash or credit. Credit means promising a payment p in numeraire in the next CM, but this is hindered by limited commitment. For credit to work, monitoring and recording is needed, to punish those who renege.

As in Gu et al. (2012, 2013), reneging is observed/recorded imperfectly: only with some probability can renegers be punished. We consider different punishments, but as a benchmark, those who are caught are forced into future autarky.

As in Kehoe and Levine (1993), this puts endogenous restrictions on debt. Lack of commitment also applies to taxes, so agents can renege on public obligations, like private obligations, with deviators similarly detected and punished. As in Andolfatto (2007), this puts endogenous restrictions on policy. This completes the description of the basic environment. We emphasize that it is based on an off-the-shelf framework in monetary economics, and was in no way rigged to get particular results. See the above-mentioned surveys for a long list of papers using versions of this model.

3 Exogenous Policy and Debt Limits

Here we define equilibrium and discuss its properties when policy and debt limits are exogenous. This may be of interest in its own right; it is certainly a useful step toward endogenizing these limits.

3.1 The CM Problem

The state of an agent of type $j = b, s$ (buyer or seller) in the CM is his net worth, $A_j = \phi m - d - T_j$, where ϕ is the value of money in terms of numeraire x, d is debt and T_j is a tax. We assume $T_s = 0$, so only buyers pay taxes if $T_b > 0$ or get transfers if $T_b < 0$ (this is only to ease a few calculations). Debt is always paid off in the current CM, without loss of generality: if debt limits are slack, agents are indifferent between short- and long-term debt; if these limits are binding, they strictly prefer the former, at least assuming interior solutions. So the state of an agent at the beginning of the DM is his real balances, ϕm. The value functions in the CM and DM are $W_j(A)$ and $V_j(\phi m)$, focusing on stationary outcomes where real variables are constant. In particular, stationarity implies ϕ falls as M grows, so real balances ϕM are constant and inflation is $\phi/\phi_{+1} = 1 + \mu$.

In the CM, the problem for an agent of type j is

$$W_j(A) = \max_{x,\ell,\hat{m}} \left\{ U^j(x,\ell) + \beta V_j(\phi_{+1}\hat{m}) \right\} \text{ st } A + w(1-\ell) = x + \phi\hat{m} \tag{2}$$

Let $x(A)$, $\ell(A)$ and $\hat{m}(A)$ be a solution. These satisfy the FOC's

$$\begin{aligned} -wU_1^j(x,\ell) + U_2^j(x,\ell) &= 0 & (3) \\ A + w(1-\ell) - \phi\hat{m} &= x & (4) \\ -\phi U_1^j(x,\ell) + \beta\phi_{+1}V_j'(\phi_{+1}\hat{m}) &\leq 0, \ = \text{ if } \hat{m} > 0. & (5) \end{aligned}$$

Obviously $\hat{m} = 0$ for sellers, since they have no use for cash in the DM. For buyers, $\hat{m} = \hat{m}_b > 0$ in monetary equilibrium (monetary equilibrium is defined formally below; for now, read it to mean a situation with $\phi M > 0$).

The following results, all based on Assumption 1 and are proved in the Appendix, greatly simplify the subsequent analysis:[5]

Lemma 1 *Given an interior solution for $x(A)$ and $\ell(A)$, $\hat{m}_j'(A) = 0$.*

Corollary 1 *Let $\Lambda_j(A) = U_1^j[x(A), \ell(A)]$. Then $W_j'(A) = \Lambda_j'(A) = 0$.*

Corollary 2 *Let $U_0^j = U^j[w - \ell(0)w, \ell(0)]$. Then $U^j[w - \ell(A)w + A, \ell(A)] = U_0^j + \Lambda_j A$.*

Lemma 1 implies the distribution of cash in the DM across buyers is degenerate, since they all take the same $\hat{m}_j$ out of the CM, independent of the amount they brought in. Hence, we do not have to track a distribution as a state variable. The corollaries say CM payoffs are linear in A. Without loss of generality, in what follows, we normalize $\Lambda_b = \Lambda_s = 1$.

[5] Our Lemma 1 was proved earlier by Wong (2012), who also characterizes the class of utility functions for which the assumption $|U^j| = 0$ holds, using different methods. This makes his argument more intricate, and so we include in the Appendix our very simple proof.

3.2 The DM Problem

The next step is to derive a simple representation of the DM problem. To begin, a buyer meets a seller in the DM with probability α, at which point they choose a quantity q and payment p subject to the constraint $p \leq L$, where $L = \phi m + D$ is the *liquidity position* of the buyer, defined as his debt limit plus real balances. To determine the terms of trade, we use a *general trading mechanism* $\mathbf{G} : \mathbb{R}^1 \to \mathbb{R}^2$, giving a pair (p, q) for any L. To understand this, consider the buyer's and seller's ex post (when they meet in the DM) trading surpluses,

$$\begin{aligned} S_b &= u(q) + W_b(A_b - p) - W_b(A_b) = u(q) - p && (6) \\ S_s &= -v(q) + W_s(A_s + p) - W_s(A_s) = p - v(q), && (7) \end{aligned}$$

using the Corollaries to Lemma 1 and the normalization $\Lambda_b = \Lambda_s = 1$. Clearly S_b and S_s do not depend on the buyer's or seller's wealth, but can depend on L due to the constraint $p \leq L$.

Hence, the terms of trade generally depend on the liquidity position of the buyer: $p = G_p(L)$ and $q = G_q(L)$. Define the efficient quantity q^* by

$$u'(q^*) = v'(q^*), \tag{8}$$

and let $p^* = \inf\{L : G_q(L) = q^*\}$ be the minimum payment required for a buyer to get q^*. To guarantee $q^* \in (0, \bar{q})$ exists, where $\bar{q}$ is a natural upper bound, we assume the gains from DM trade are positive but finite:

Assumption 2 *$u'(0) > v'(0)$ and $\exists \bar{q} > 0$ such that $u(\bar{q}) = v(\bar{q})$.*

Given q^*, we focus on mechanisms of the form

$$G_p(L) = \begin{cases} L & \text{if } L < p^* \\ p^* & \text{otherwise} \end{cases} \quad \text{and } G_q(L) = \begin{cases} g(L) & \text{if } L < p^* \\ q^* & \text{otherwise} \end{cases} \tag{9}$$

where g is some strictly increasing function with $g(0) = 0$ and $g(p^*) = q^*$.

This class of mechanisms includes competitive pricing, as used in most of the credit literature, and bargaining, as used in much of monetary economics. Section 4 presents four simple and natural axioms that, when satisfied, imply $\mathbf{G}$ must take the form in (9), and discusses several common special cases. For now we focus on the economic content: (9) says that a buyer gets the efficient quantity q^* and pays some amount p^*, determined by the mechanism, if he can afford it, in the sense that $p^* \leq L$; and when he cannot afford that, he pays $p = L < p^*$ and gets $q = g(L) < q^*$. Thus, $g(L)$ indicates the quantity a constrained buyer gets. Inversely, if $f = g^{-1}$ then $p = f(q)$ is the cost of q. Again, Section 4 shows how competitive pricing, different bargaining solutions and some more abstract mechanisms can be seen as special cases of our general class of mechanisms, but all we need for now, for technical convenience, is that g and f are twice continuously differentiable almost everywhere.

Now consider a seller, who as we said takes $\hat{m} = 0$ to the DM. If he does not trade, he gets continuation value $W_s(0)$. If he trades, from (7), he gets a surplus over the continuation value $p - v(q)$, where $p = G_p\left(\bar{L}\right)$ and $q = G_q\left(\bar{L}\right)$ are determined by (9) with $\bar{L}$ denoting the liquidity position of a representative buyer. Similarly, for a buyer taking $\hat{m}$ into the DM, his expected payoff is

$$V_b(\phi_{+1}\hat{m}) = W_b\left(\phi_{+1}\hat{m} - T\right) + \alpha\left[u(q) - p\right],$$

where $p = G_p(L)$ and $q = G_q(L)$ depend on his own liquidity L. It is now easy to show (see the Appendix) the following:

Lemma 2 *In any monetary equilibrium buyers are constrained: $q < q^*$.*

In monetary equilibrium the payment $p = f(q)$ exhausts liquidity $L = D + \phi_{+1}\hat{m}$. Substituting this into V_b, then substituting that into the CM value function, after simplification we get

$$W_b(A) = U_0^b + A - \beta T + \beta W_b(0) + \beta\left\{-i\phi_{+1}\hat{m} + \alpha\left[u(q) - f(q)\right]\right\}, \tag{10}$$

where i denotes the nominal interest rate, defined via the Fisher equation $1+i = (1+\mu)/\beta$.[6] Also, from the Fisher equation, it does not matter if the monetary authority sets i or μ, so we arbitrarily take i to be their policy instrument. Then, using $f(q) = \phi_{+1}\hat{m} + D$, we rewrite (10) as

$$W_b(A) = k + A + \alpha\beta J(q;i),$$

where $k = U_0^b - \beta T + \beta W_b(0) + \beta i D$ is a constant, and hence irrelevant for the choice of $\hat{m}$, and

$$J(q;i) = u(q) - (1 + i/\alpha) f(q). \tag{11}$$

This change of variable replaces buyers' choice of $\hat{m}$ with a direct choice of q. Without loss of generality we impose $q \in [0, q^*]$, and represent this choice by

$$q_i = \arg\max J(q;i) \text{ st } q \in [0, q^*]. \tag{12}$$

Clearly a solution q_i exists and satisfies $q_i < q^*$ by Lemma 2. In monetary equilibrium, $q_i > 0$ satisfies the FOC which we write as

$$e(q) \equiv u'(q) - (1 + i/\alpha) f'(q) = 0. \tag{13}$$

Given a solution to $e(q_i) = 0$, we have $p_i = f(q_i)$, and real balances are $\phi M = f(q_i) - D$. For a monetary equilibrium we need $\phi M > 0$, which we have just reduced to $D < f(q_i)$.

[6] To derive this, notice that

$$\begin{aligned} W_b(A) &= U_0^b + A - \phi\hat{m} + \beta\left\{W_b\left(\phi_{+1}\hat{m} - T\right) + \alpha\left[u(q) - f(q)\right]\right\} \\ &= U_0^b + A - \beta T + \beta W_b(0) - \phi\hat{m} + \beta\left\{\phi_{+1}\hat{m} + \alpha\left[u(q) - f(q)\right]\right\}, \end{aligned}$$

then use the Fisher equation. For our purposes, the Fisher equation is merely an accounting identity defining i, although one can also interpret i as the nominal return that makes agents indifferent to borrowing and lending between one CM and the next CM.

3.3 Equilibrium

We now define (symmetric, stationary) equilibrium formally:

Definition 1 *Given a mechanism* $\mathbf{G}$*, debt limit* D *and policy* i*, a monetary equilibrium is a nonnegative DM outcome* (p, q)*, CM allocation* $(x, \ell, \hat{m})$ *and real balances* $\phi M > 0$ *such that:*

1. q *solves (12),* $p = f(q)$ *and* $\phi M = p - D > 0$*;*

2. $(x, \ell, \hat{m})$ *solves (2) for all agents, with* $\hat{m} = 0$ *for sellers and* $\hat{m} = M_{+1}$ *for buyers, and* $\int x = \int \ell$ *(market clearing).*

Definition 2 *A nonmonetary equilibrium is similar except* $\phi M = 0$*.*

As in any good model of fiat currency, there is always a nonmonetary equilibrium, and here it is straightforward to check that it exists uniquely. As regards monetary equilibria, note that (q, p) can be determined independently of (x, ℓ), and therefore one can discuss some properties of the DM outcome without reference to the CM allocation.[7] Since the CM equilibrium allocation is easily analyzed using standard methods, we do not dwell on it, and instead concentrate on (p, q) and the value of money ϕ. The strategy is this: first look for a solution $q_i > 0$ to (12); if $p_i = f(q_i) > D$ then $\phi M > 0$ and a monetary equilibrium exists; and if $f(q_i) \leq D$ then monetary equilibrium does not exist, and the only equilibrium is nonmonetary.

To insure the solution to (12) is $q_i > 0$, impose:

[7] This dichotomy, which obtains because the FOC's (3)-(4) can be solved for (x, ℓ) independent of the DM outcome, obviously depends on q entering $\mathcal{U}^s$ and $\mathcal{U}^s$ separably in (1). This simplifies the analysis but we do not think it is critical for the economic insghts. It is well known that one can break the dichotomy by interacting CM and DM variables in tastes or technology, or via financial considerations, as in Venkateswaran and Wright (2013); more on this below.

Assumption 3 *$\exists q > 0$ such that $f(q) < u(q)$.*

This holds for most interesting mechanisms, including Walrasian pricing, and Nash or Kalai bargaining with buyer bargaining power $\theta > 0$. Given Assumption 3, in the limit as $i \to 0$ we have $q_0 = \arg\max_q J(q;0) > 0$. Hence, $q_i > 0$ at least for i not too big. In the Appendix we also prove the following:[8]

Lemma 3 *For generic i, the solution q_i to (12) is unique and satisfies $\partial q/\partial i < 0$.*

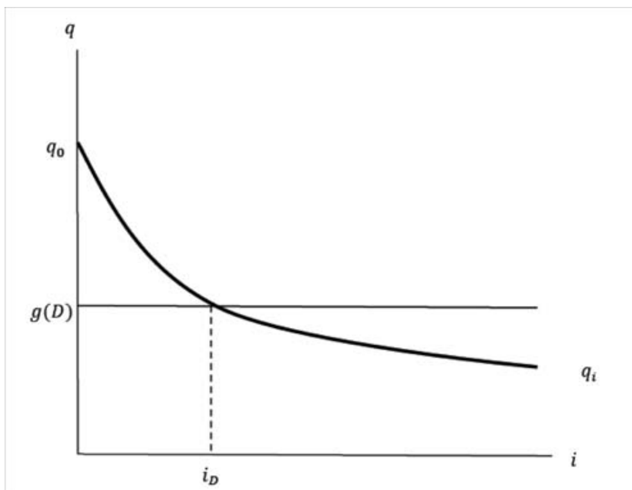

Figure 1: KM Quantity q vs Nominal Rate i

Figure 1 shows q_i versus i. The intercept is $q_0 \leq q^*$, since we can never have $q > q^*$.[9] For generic i, q_i is single-valued and strictly decreasing. As we said, we

[8]Lemma 1 is based on Wright (2010); we include the details in the Appendix because the environments are somewhat different, and because there is a technical problem in the original argument. As discussed in the proof, as i crosses some nongeneric value, say $\tilde{\imath}$, q_i may jump discretely downward because there are multiple solutions to $\max_q J(q;0)$. We can rule that out for some mechanisms, including Walrasian pricing and Kalai bargaining, which make J stricly concave, but whether or not q_i jumps the substantive results are the same. Similarly, while Lemma 3 says monetary equilibrium is generically unique, that is not relevant for the main substantive conclusions: if multiple monetary equilibria exist, the results about credit apply to all of them.

[9]This is clear from (9), and basically follows directly from Axiom 4 in Section 4. By way of examples, one can check that $q_0 = q^*$ with Walrasian pricing or Kalai bargaining, but $q_0 < q^*$ with Nash bargaining unless $\theta = 1$.

need $f(q_i) > D$, or equivalently $q_i > g(D)$, to have $\phi M > 0$. Given a D such that $g(D) < q_0$, as in Figure 1, there is a unique $i_D > 0$ such that monetary equilibrium exists iff $i < i_D$. Or, to characterize the outcome in terms of D, first suppose $D \geq f(q^*)$. Then buyers can get q^* on credit, and if we try to construct a monetary equilibrium we fail, because $\phi M = f(q_i) - D \leq f(q^*) - D \leq 0$. Second, suppose $f(q^*) > D \geq f(q_i)$. Then buyers can only get $q = g(D) < q^*$ on credit, but if we try to construct a monetary equilibrium we still get $\phi M = f(q_i) - D \leq 0$. Finally, suppose $f(q_i) > D$. Then $\phi M = f(q_i) - D > 0$ and monetary equilibrium exists. Hence, we have proved:

Proposition 1 *There are three possible outcomes:*

1. *if $D < f(q_i)$ there is a unique nonmonetary equilibrium with $q = g(D) < q_i$ and generically there is a unique monetary equilibrium with $q = q_i$;*
2. *if $f(q_i) \leq D < f(q^*)$ there is no monetary equilibrium and there is a unique nonmonetary equilibrium with $q \in [q_i, q^*)$;*
3. *if $f(q^*) \leq D$ there is no monetary equilibrium and there is a unique nonmonetary equilibrium with $q = q^*$.*

Now notice something interesting: in monetary equilibrium $q = q_i$ does not depend on D. This is because buyers acquire real balances up to the point where the marginal benefit equals i, or $e(q_i) = 0$ as given in (13). This means the value of money and hence real balances adjust to guarantee the liquidity provided by cash fills the gap between the desired level of L and the debt limit, $\phi M = f(q_i) - D$. This is not to say that an individual's debt limit is irrelevant: if we keep everyone else the same and lower D for one agent he may well be worse off; but if we lower D for everyone then ϕM adjusts to keep L the same.

To argue the same point slightly differently, first compute welfare for a representative buyer in monetary equilibrium, at the initial date $t = 0$, with $m = M$ dollars and tax obligation T:[10]

$$W_0 = W_b\left(\phi M - T\right) = \frac{U_0^b + \alpha\beta\left[u\left(q\right) - f\left(q\right)\right]}{1-\beta}. \tag{14}$$

Since D vanishes from (14), the debt limit simply does not matter. Hence, we conclude this: When D is small *money is essential*, because $D < f\left(q_i\right)$ implies there is a monetary equilibrium with $q_i > g\left(D\right)$, so welfare can be higher with than without money. But in any monetary equilibrium, *credit is not essential* and *changes in credit conditions are neutral.* Of course, if $f\left(q^*\right) > D > f\left(q_i\right)$ then $q = g\left(D\right)$ depends on D, but then the equilibrium is nonmonetary.

Summarizing these observations, we have:

Proposition 2 *Monetary equilibrium exists and money is essential iff $D < f\left(q_i\right)$. In this case both money and credit may be used, but credit is inessential, and changes in D are neutral.*

These results may be surprising. One might think higher D allows agents to increase x or ℓ, since they can cut back on their real balances and be equally liquid. That is incorrect. The decline in desired real balances after an increase in D is exactly offset by the reduction in the value of the currency one is currently holding – i.e., there is *complete crowding out* of money by credit. Of course, if buyers had heterogeneous endowments of m at $t = 0$, since D affects ϕ, it affects

[10] To see this, note that is

$$\begin{aligned} W_b\left(\phi M - T\right) &= U_0^b + \phi M - T - \phi\hat{m} + \beta\left\{W\left(\phi_{+1}\hat{m} - T\right) + \alpha\left[u\left(q\right) - f\left(q\right)\right]\right\} \\ &= U_0^b + \beta\left\{W\left(\phi_{+1}\hat{m} - T\right) + \alpha\left[u\left(q\right) - f\left(q\right)\right]\right\}, \end{aligned}$$

using $T = -\mu\phi M$ (the government budget) and $\hat{m} = \left(1+\mu\right)M$ (market clearing). Since $W\left(\phi_{+1}\hat{m} - T\right) = W_0$ in a stationary monetary equilibrium, this reduces to (14).

the distribution of wealth, but this effect vanishes after $t = 1$ after agents visit the CM. This seems similar to standard neutrality results for money that say changes in M matter if they redistribute wealth, but not if they don't. We return to this in Section 5.

4 Mechanisms

Here we digress by discussing trading mechanisms in more detail. Again, we want to use a general class of mechanisms to show the results do not depend on a particular way of determining the terms of trade. How general is our class of mechanisms? Consider imposing the following natural axioms:

Axiom 1 *(Feasibility):* $\forall L,\ 0 \leq G_p(L) \leq L,\ 0 \leq G_q(L)$.

Axiom 2 *(Individual Rationality):* $\forall L,\ u \circ G_q(L) \geq G_p(L)$ *and* $G_p(L) \geq v \circ G_q(L)$.

Axiom 3 *(Monotonicity):* $G_p(L_2) > G_p(L_1) \Leftrightarrow G_q(L_2) > G_q(L_1)$.

Axiom 4 *(Bilateral Pareto Efficiency):* $\forall L, \nexists (p', q')$ *with* $p' \leq L$ *such that* $u(q') - p' > u \circ G_q(L) - G_p(L)$ *and* $p' - v(q') > G_p(L) - v \circ G_q(L)$.

Note Axiom 3 does not say that the surpluses S_b and S_s are increasing in L, only that one must pay more to get more (so, e.g., generalized Nash bargaining satisfies this, even though S_b need not to be increasing in L, as discussed in Aruoba et al. 2007). Also, although Axiom 4 seems reasonable, it is not critical for the main results, as credit conditions can also be neutral with inefficient mechanisms (see below). And note that Axiom 4 is an ex post condition, saying we cannot make the parties better off conditional on the buyer's liquidity L; it does not say the ex ante choice of L is efficient.

The following result is proved in the Appendix:

Proposition 3 *Any mechanism* $\mathbf{G}$ *satisfying A1-A4 implies (9).*

A simple example is Kalai's proportional bargaining mechanism, with θ denoting the buyer's bargaining power. Kalai's solution in this context maximizes S_b st $S_b = \theta(S_b + S_s)$ and $p \leq L$. Recalling q^* from (8), let $p^* = f(q^*)$ where

$$f(q) = \theta v(q) + (1-\theta) u(q). \tag{15}$$

It is easy to check Kalai's bargaining solution boils down to this: if $p^* \leq L$ then $(p,q) = (p^*, q^*)$; and if $p^* > L$ then $p = L$ and $q = g(L)$ with $g = f^{-1}$ with f given by (15). Consistent with (9), when the liquidity constraint is slack q maximizes the joint surplus and p splits it between the parties according to θ, but when the liquidity constraint binds, $p = L$ and q splits the surplus.

Generalized Nash bargaining, where (p,q) maximizes $S_b^\theta S_s^{1-\theta}$ st $p \leq L$, gives a similar outcome except now

$$f(q) = \frac{\theta u'(q) v(q) + (1-\theta) v'(q) u(q)}{\theta u'(q) + (1-\theta) v'(q)}. \tag{16}$$

The Nash and Kalai solutions are the same if $\theta = 1$ or if $u(q) = v(q) = q$. When $\theta < 1$ and $u'' < 0$ or $v'' > 0$, they are different when $p \leq L$ binds. But qualitatively they are the same and both take the form specified in (9).

Some search-based monetary models use Walrasian pricing (see Rocheteau and Wright 2005 for a discussion). This gives rise to the same qualitative outcome, but now $f(q) = Pq$, where buyers take P as beyond their control, although in equilibrium $P = v'(q)$. An example that does not correspond to (9) is a monopsonist that takes as given the supply curve $P = v'(q)$ rather than the price:

$$\max_q \{u(q) - qv'(q)\} \text{ st } qv'(q) \leq L. \tag{17}$$

Let $\tilde{q}$ be a solution without the liquidity constraint – i.e., the standard monopsonist solution – and let $\tilde{p} = \tilde{q}v'(\tilde{q})$. This mechanism looks like (9), except the critical value is $\tilde{p} = f(\tilde{q})$ rather than $p^* = f(q^*)$. Still, it is easy to see that the substantive results (e.g., the neutrality of credit conditions) hold for the monopsony mechanism.

In terms of less standard solution concepts, following Hu et al. (2009), consider trying to construct $f(q)$ so that equilibrium supports a desirable q^o, which could be q^*, or something else. This is important for our purposes because we want results that hold even when we try our best to support good outcomes with creatively designed mechanisms. In particular, given i, sometimes we can achieve good outcomes using only money and sometimes we cannot. We want to know how well we can do using only money, so that results on the inessentiality and neutrality of credit are not misunderstood as depending on the ability to achieve optimal outcomes using only money. Here is the main result on this dimension:

Proposition 4 *Let $\hat{q}$ solve $u(\hat{q}) = (1 + i/\alpha) v(\hat{q})$. Then there exists a mechanism to support any $q^o \leq \min\{q^*, \hat{q}\}$. In particular, if $q^* \leq \hat{q}$ we can support $q_i = q^*$ even if $i > 0$.*

The proof, which constructs the mechanism explicitly, is in the Appendix.[11] The idea is straightforward. We want the terms of trade – i.e., the function $f(q)$ – to give buyers the incentive ex ante, in the CM, to bring the right amount of money to the DM. Recall the objective function $J(q, i) = u(q) - (1 + i/\alpha) f(q)$. For buyers to bring enough cash to get q^o we need $u'(q^o) = (1 + i/\alpha) f'(q^o)$. But we also have to be sure that agents have the incentive to go through with trade ex post, in the DM, so it has to satisfy Axioms 2 and 4. There are various ways

[11] Although our mechanism is very much in the spirit of Hu et al. (2009), since the details are quite different, we provide the proof.

to construct an $f(q)$ to accomplish this; we provide one that is continuous, and linear as long as ex post incentive conditions are slack. Still, some q^o cannot be supported if i is too high, since that makes the $\hat{q}$ in Proposition 4 small. In particular, when β is low we cannot achieve q^* unless i is low. Hence, it can be important to reduce i, which is why we are interested in endogenous policy limits.

5 Endogenous Debt and Deflation Limits

Assume now that when the time comes to repay d buyers can renege. As in Kehoe and Levine (1993), some punishment is needed to make credit viable. For money to be essential, we will show that the monitoring of repayment cannot be perfect, a result very much in the spirit of Kocherlakota (1998). At the same time, for credit to be essential, it must be the case that desirable outcomes cannot be achieved using money only. For many solution concepts, including Kalai bargaining and Walrasian pricing, one can show $q_i = q^*$ iff $i = 0$. Why not run the Friedman rule and be done with it? An answer proposed by Andolfatto (2007) is that commitment problems can render $i = 0$ infeasible. Now Proposition 4 shows it may be possible to design a mechanism to a desirable q^o even when $i > 0$, but only if $q^o < \hat{q}$, and big i makes $\hat{q}$ small. Hence, it is of interest to know what values of i are feasible.

Note $i < r$ requires decreasing prices, and hence negative growth in the money supply, and hence taxation, $T > 0$. Andolfatto's (2007) point is that individuals may fail to pay T if it is too high, even at risk of punishment. This is nice, from our vantage, because the same frictions that hinder credit, including imperfect monitoring, hinder the ability to tax, and hence to achieve a low nominal interest rate, and hence to achieve desirable outcomes using money only. As a special case, if we cannot monitor tax compliance at all then we cannot have any deflation, just like we cannot have any credit when we cannot monitor repayment. The goal is to determine generally how much debt and deflation we can have when monitoring is possible but not perfect.

To this end, we need to specify the sequence of events in the CM precisely. First, a buyer decides whether to pay d, which is monitored with probability δ. If he pays d, or does not pay but is not caught, he then decides whether to pay T, which is monitored (or audited) with probability τ. If he pays T, or does not pay but is not caught, he chooses x, ℓ and $\hat{m}$ as before. Deviators that are caught, debt defaulters and tax evaders, are reduced to autarky.[12] In autarky agents produce x for themselves with ℓ, pay no CM taxes and get no CM transfers. Although they are banned from future markets, we let them spend any cash on hand in the period in which they first deviate. The autarky payoff is therefore

$$\begin{aligned}\underline{W}(\phi m) &= \max_{x,\ell}\left\{U^b(x,\ell)+\beta\underline{W}(0)\right\} \text{ st } \phi m + w(1-\ell) = x \\ &= \frac{1+r}{r}U_0+\phi m,\end{aligned}$$

using Lemma 1 and Corollary 2, plus the obvious result that anyone excluded from the DM chooses $\hat{m}=0$.

Working backwards from the tax payment decision, compliance at this stage requires[13]

$$W_b(\phi m - T) \geq \tau\underline{W}(\phi m) + (1-\tau)W_b(\phi m).$$

The LHS is the continuation value from paying taxes; the RHS is the expected payoff to not paying, where there is a chance τ of getting caught and a chance $1-\tau$ of getting away with it. Inserting W_b and $\underline{W}$, we get the tax payment

[12] The Appendix considers an alternative punishment scenario, where deviators can continue trading in the DM but only using cash – i.e., what they lose is access to credit.

[13] Without loss in generality, we restrict attention to one-shot deviations; for our purposes this is simply the unimprovability principle of dynamic programming.

constraint

$$T \leq \frac{\tau}{r+\tau} \{\alpha [u(q) - f(q)] - i\phi M\}. \tag{18}$$

Or, using the government budget equation $T = -\mu\phi M$ and the Fisher equation,

$$\alpha [u(q) - f(q)] \geq \frac{r [r + \tau - i(1-\tau)] \phi M}{(1+r)\tau}. \tag{19}$$

If money growth μ is nil or inflationary $(i \geq r)$ then $T \leq 0$ and (19) is satisfied trivially. If monetary policy is deflationary $(i > r)$, however, it must respect this limit to deflation (19), which generally entails a lower bound for i.

Definition 3 *A policy i is feasible if a monetary equilibrium exists and (19) is satisfied.*

Moving to the incentive condition for honoring private obligations, we get the debt payment constraint

$$W_b(\phi m - T - d) \geq \delta \underline{W}(\phi m) + (1-\delta) W_b(\phi m - T),$$

where there is a chance δ of getting caught and a chance $1-\delta$ of getting away with it. Using the government budget equation, this reduces to

$$d \leq \frac{\delta}{r} \{\alpha [u(q) - f(q_i)] - r\phi M\}. \tag{20}$$

Note that for any feasible policy, in monetary equilibrium, the RHS of (20) is strictly positive.[14]

We now endogenize the debt limit, using a version of the method in Alvarez and Jermann (2000), generalized to incorporate money: Start with an arbitrary D. The RHS of (20) depends on this D, since in general q and ϕM do. From Proposition 1, the RHS of (20) can be written:

[14]For inflation, the RHS is proportional to $J(q; i) + (i - r)\phi M > 0$; for defaltion, use (19).

$$
\Phi(D) \equiv \begin{cases} \dfrac{\delta\alpha}{r}\left[u(q_i) - (1 + r/\alpha) f(q_i)\right] + \delta D & \text{if } D < f(q_i) \\ \dfrac{\delta\alpha}{r}\left[u \circ g(D) - D\right] & \text{if } f(q_i) \leq D < f(q^*) \\ \dfrac{\delta\alpha}{r}\left[u(q^*) - f(q^*)\right] & \text{if } f(q^*) > D \end{cases} \tag{21}
$$

This assumes that we select the monetary equilibrium when it exists, i.e., in the first branch, where $D < f(q_i)$. We can also select the nonmonetary equilibrium, in which case the first branch is the same as the second, $(\delta\alpha/r)\left[u \circ g(D) - D\right]$. See Figure 2, where the solid curve is drawn selecting the monetary equilibrium when it exists, and the dashed curve is drawn selecting the nonmonetary equilibrium over the same region.

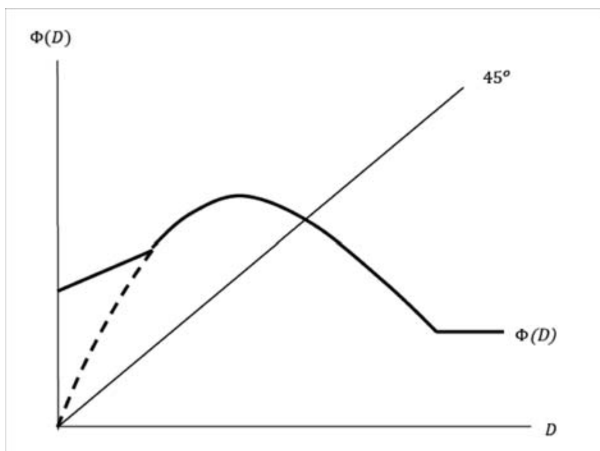

Figure 2: The Correspondence $\Phi(D)$.

Given D, the repayment constraint is satisfied iff $d \leq \Phi(D)$. That is, if the debt limit is exogenously set to D, agents would be willing to honor obligation d iff it were less than $\Phi(D)$. Therefore, we have:

Definition 4 *An endogenous debt limit is a nonnegative fixed point $\hat{D} = \Phi(\hat{D})$.*

Consider first selecting the nonmonetary equilibrium when $D < f(q^*)$. Then

Φ is single-valued, continuous and constant when D is large:

$$\Phi(D) = \Phi^* \equiv \frac{\delta\alpha}{r}\left[u(q^*) - f(q^*)\right] \ \forall D \geq f(q^*).$$

Since $\Phi(0) = 0$, one endogenous debt limit is $\hat{D} = 0$, where the DM shuts down (given we have selected the nonmonetary equilibrium). In this situation, you believe there will be no DM credit (and no monetary exchange) in the future, so you would renege on any $d > 0$ in the present, since exclusion from the market is painless. Hence $\hat{D} = 0$ is an endogenous debt limit. There coexist others if $\Phi'(0) > 1$, which is true if $u'(0)$ is big. Hence, if $u'(0)$ is big there is at least one $\hat{D} = \Phi(\hat{D}) > 0$, where you believe the debt limit is positive, so exclusion from the market is painful, and that makes $\hat{D}$ self enforcing.[15]

Consider now selecting the monetary equilibrium when $D < f(q_i)$. This does *not* mean the economy will end up in a monetary equilibrium. It only says that we would be in a monetary equilibrium if D were low, but the endogenous $\hat{D}$ may not be low. When D is low, the first branch of $\Phi(D)$ is linear with slope δ and intercept

$$\Phi(0) = (\delta\alpha/r)\left[u(q_i) - (1 + r/\alpha) f(q_i)\right].$$

For $i \geq r$, we have $\Phi(0) > (\delta\alpha/r) J(q_i; i) > 0$. For $i < r$, we have $\Phi(D) > 0$ $\forall D > 0$ if i satisfies (19), and so we have $\Phi(0) > 0$ for feasible policies. Hence, $D = 0$ is not a fixed point. Intuitively, if money is valued, you would honor some $d > 0$ to avoid expulsion from the market, even if there were no credit, since you can use cash.[16]

[15] In case it is not obvious, if $\Phi(0) = 0$ and $\Phi'(0) > 0$, since $\Phi(D) = \Phi^*$ for large D there exists $\hat{D} = \Phi(\hat{D}) > 0$ by continuity. There can be multiple positive fixed points, and $\hat{D}$ can be above or below $f(q^*)$, so the debt limit may or may not bind. See Gu et al. (2012*b*) for more on endogenous debt limits, including nonstationary outcomes, in nonmonetary economies; to focus is on the interaction between money and credit, from now on we select the monetary equilibrium when it exists.

[16] One might say that money is complementary with credit: selecting the monetary equi-

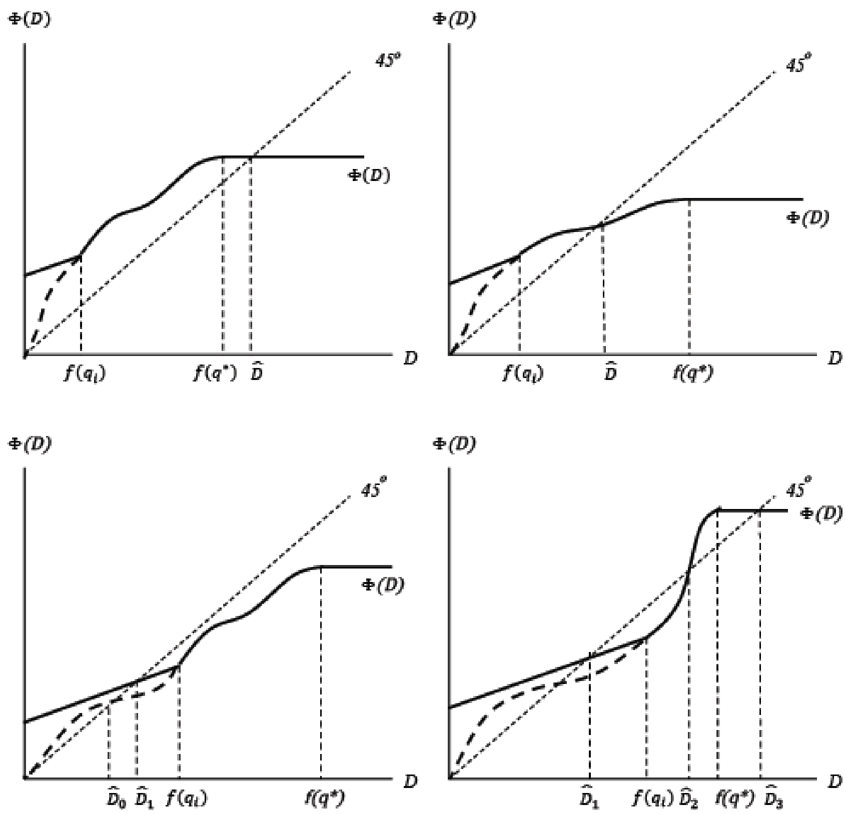

Figure 3: Endogenous Debt Limits

Since $\Phi(0) > 0$ and $\Phi(D) = \Phi^*$ for large D, there always exists an non-degenerate endogenous debt limit, $\hat{D} = \Phi(\hat{D}) > 0$. Given a fixed point $\hat{D}$, to see if it is consistent with monetary equilibrium, as always we must check if $\phi M = f(q_i) - \hat{D} > 0$. If so, the endogenous $\hat{D}$ is on the linear branch of Φ. Figure 3 shows versions of Figure 2 in several cases: the first panel has $\hat{D} > f(q^*)$, which means the debt limit is not binding; the second has $f(q^*) > \hat{D} > f(q_i)$, so the debt limit is binding but money is not valued; the third has $\hat{D} < f(q_i)$, so there is a monetary equilibrium; and the fourth shows multiple endogenous debt

librium for low D precludes the $\hat{D} = 0$ as an endogenous debt limit. This is counter to some results, where valued money is bad for credit (e.g., Berentsen et al. 2007; Aiyagari and Williamson 1999), because in those models the punishment for deviators is trade using money, while here it autarky.

limits, one of each type. We summarize these results as follows:

Proposition 5 *Given an incentive feasible policy i, an endogenous debt limit $\hat{D} = \Phi(\hat{D}) \geq 0$ exists. The possible outcomes are:*

1. *if $\hat{D} < f(q_i)$ there is a monetary equilibrium with $q = q_i$;*
2. *if $f(q_i) \leq \hat{D} < f(q^*)$ there is no monetary equilibrium and there is a nonmonetary equilibrium with $q \in [q_i, q^*)$.*
3. *if $f(q^*) \leq \hat{D}$ there is no monetary equilibrium and there is a nonmonetary equilibrium with $q = q^*$.*

While the multiplicity of endogenous debt limits may be interesting, one can also give conditions to rule it out. The following is not hard to show:

Proposition 6 *Suppose $S_b(D) = u \circ g(D) - D$ is concave, which is true for Walrasian pricing and Nash or Kalai bargaining with θ close to 1. Then given a feasible policy, there is a unique positive endogenous debt limit $\hat{D} > 0$.*

Now that endogenous debt limits are understood, let us return to conditions for a policy i to be feasible. First, in monetary equilibrium with an endogenous debt limit we can solve $\Phi(D) = D$ explicitly as

$$\hat{D}_i = \frac{\delta}{1-\delta}\frac{\alpha}{r}\left[u(q_i) - (1 + r/\alpha) f(q_i)\right], \tag{22}$$

where we indicate here that $\hat{D}_i$ depends on i. Recall that when there were no constraints on policy (taxes were exogenously enforceable), a monetary equilibrium existed iff $f(q_i) > \hat{D}$; now we also need to check $\hat{D}$ satisfies the feasibility condition (19). Thus, to get a monetary equilibrium, it suffices to show $\hat{D} < f(q_i)$ and (19) holds.

From (22), $D < f(q_i)$ iff $H(q_i) < 0$, where $H(q) \equiv u(q) - (1 + r/\alpha\delta) f(q)$. Also, (19) always holds for $i \geq r$, and so with a constant or increasing price level $H(q_i) < 0$ is the only condition for monetary equilibrium. For deflation, however, a calculation indicates that (19) holds iff $K(q_i; i) \geq 0$, where $K(q; i) \equiv u(q) - (1 + \Omega_i) f(q)$ and

$$\Omega_i = \frac{r}{\alpha} \frac{\tau + r - (1-\tau) i}{\tau + [\delta + (1-\delta)\tau] r - \delta (1-\tau) i}.$$

Hence, for deflationary policies, there exists a monetary equilibrium iff satisfies $H(q_i) < 0 \leq K(q_i; i)$. Summarizing:

Proposition 7 *Given $i > 0$, consider a candidate monetary equilibrium, with q_i solving (13), and an endogenous debt limit $\hat{D}_i$ given by (22). Then we have:*

1. *if $H(q_i) \geq 0$ then i is not feasible as it is inconsistent with monetary equilibrium;*

2. *if $H(q_i) < 0$ then monetary equilibrium exists and i is feasible iff (2a) $i \geq r$ or (2b) $i < r$ and $K(q_i; i) \geq 0$.*

Proposition 7 reduces in an important case to Kocherlakota's (1998) result that says money cannot be essential when there is perfect monitoring of debt repayment. To be precise, (22) implies $\hat{D}_i \to \infty$ as $\delta \to 1$. Hence, for perfect monitoring, the endogenous debt limit must exceed $f(q_i)$, and hence money cannot be valued. When $\delta = 1$, if one considers monetary equilibrium in Kocherlakota's setup, one finds that the planner does no worse using credit only; when $\delta = 1$ here, one finds that the market debt limit is sufficiently big that in equilibrium agents do no worse using only credit, and hence do not value money Summarizing:

Proposition 8 *If $\delta = 1$, with endogenous debt limits, monetary equilibrium does not exist.*

It is hard to characterize the set of i's that satisfy the conditions in Proposition 7, in general, but if $K(q; i)$ and $H(q)$ are concave in q one can say more.[17] Concavity of K means there is a unique $\bar{q}_i > 0$ with $K(\bar{q}_i; i) \gtreqless 0$ as $\bar{q}_i \gtreqless q_i$. At $i = r$, we know $q_i < \bar{q}_i$ because $u(q_r) > (1 + r/\alpha) f(q_r)$ in monetary equilibrium. As i decreases, q_i increases and $\bar{q}_i$ decreases, so there is a unique $\underline{i}$ where they meet. Concavity of $H(q)$ means there is an upper bound $\bar{i}$ such that $i \gtreqless \bar{i}$ as $H(q) \gtreqless 0$. If $\underline{i} < \bar{i}$, then $i < r$ is feasible, and $\underline{i}$ is a lower bound on i, which may or may not be positive. If $\underline{i} > 0$ then the Friedman rule is infeasible. A calculation implies that the Friedman rule is feasible if $H(q_0) < 0 \leq K(q_0; 0)$, which reduces to

$$(1 + r\Omega_0/\alpha) f(q_0) < u(q_0) < (1 + r/\alpha\delta) f(q_0).$$

An example is shown in Figure **??**. The left panel depicts $H(q)$ and $K(q)$, as well as $e(q)$ from (13), the 0 of which is potentially a monetary equilibrium. It is drawn using Kalai bargaining with $\theta = 0.85$, and other parameters set to $r = \alpha = 0.1$, $\rho = 1/2$ and $\tau = 1$, which is convenient because $\tau = 1$ implies K is independent of i (as in H for any τ). For $i = 0.04$ the solution to $e(q) = 0$ is such that $H(q) < 0 < K(q)$, and so this q constitutes a monetary equilibrium. When we try to lower i to $i' = 0.01$, the solution to $e(q) = 0$ violates $0 < K(q)$ – which means households are unwilling to pay the taxes required to decrease i and increase q that much.

For another example, the right panel shows the relationship between i and

[17] We cannot show $K(q; i)$ is concave either, in general, but we can in some cases. With Kalai bargaining, e.g., $K(q; i)$ is concave for θ close to 1; for a general θ it is concave for σ close to 1 when $u(q) = q^\sigma$.

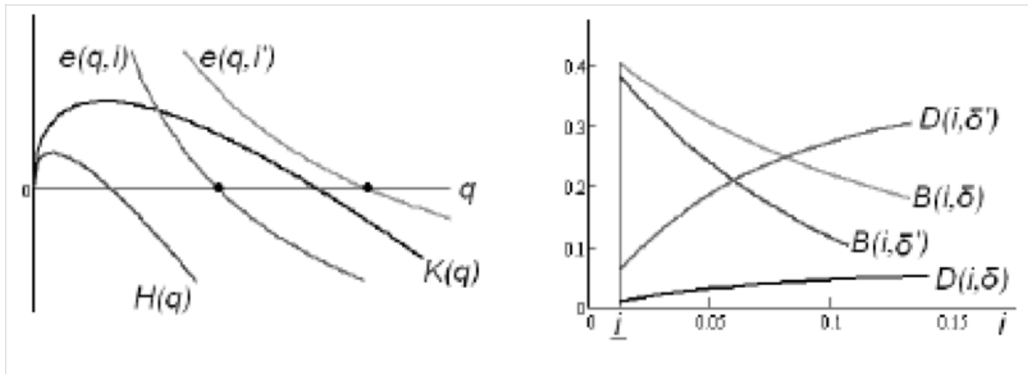

Figure 4: Endogenous Debt and Deflation Limits

two endogenous variables: the debt limit D; and a standard measure of money demand, $B = \phi M/Y$, where $Y = \alpha f(q) + x$ is output, with $x = A = 2$ generated by $U(x, \ell) = A \log(x) + \ell$. The other parameters are $r = \alpha = 0.25$, $\tau = 1$, and either $\delta = 0.1$ or $\delta' = 0.4$. When δ is bigger the debt limit $D(i, \delta)$ increases, naturally, while money demand falls, although in other examples the $B(i, \delta)$ and $B(i, \delta')$ curves could cross. Also, here $D(i, \delta)$ is monotone in i, but that was not always the case. Note that as i gets larger, money demand eventually hits $B(i, \delta) = 0$, so monetary equilibrium breaks down, because $H(q) < 0$ is violated. In this situation, agents abandon cash and use only credit. Monetary equilibrium can also break down for small i, because $K(q) > 0$ is violated, so agents stop paying taxes. Hence, monetary equilibria requires i is neither too high nor too low.

The bigger point, however, is this: The main result with exogenous policy and debt limits – that as long as we are in monetary equilibrium credit is inessential and changes in credit conditions are neutral – also holds with endogenous deflation and debt limits. This it is not because we can run policies that achieve desirable outcomes using cash only. It is because changes in credit conditions crowd out real balances one for one. This was true when the debt limit was exogenous, and also here when it is endogenous, where changes in D can be understood to mean changes in the parameters affecting it.

6 Extensions

The next step is to investigate the robustness of the main results. We focus on the case where the debt limit D is exogenous, and the tax T is enforceable, but the methods discussed above can be used to endogenize these.

6.1 Shocks

We first introduce shocks that make q random, motivated by the old idea that individuals often use money for smaller purchases and credit for larger purchases. Could money and credit both be essential if q is sometimes big and sometimes small? To address this is a general way, let $\varepsilon_t = (\varepsilon_{1t}, \varepsilon_{2t}, ...)$ be a vector of shocks to u and v, possibly including aggregate, idiosyncratic and match-specific shocks. Let $\varepsilon_t \sim F_t(\cdot|h^t)$, where $h^t = (\varepsilon_1, \varepsilon_2, ..., \varepsilon_{t-1})$ is the history, and assume for now that ε_t is not known at $t-1$, at least not until the CM at $t-1$ is closed and people have already decided $\hat{m}_{t-t}$. When $\varepsilon_t = \varepsilon$, write u_ε and v_ε for DM utility and cost, and f_ε and g_ε for the mechanism. The efficient quantity is now given by $u'_\varepsilon(q^*_\varepsilon) = v'_\varepsilon(q^*_\varepsilon)$, and as usual we have $q = q^*_\varepsilon$ if $L \geq f_\varepsilon(q^*_\varepsilon)$ while $q = g_\varepsilon(L)$ otherwise. Let $\mathcal{U}_t(h^t) = \{\varepsilon_t = \varepsilon : L \geq f_\varepsilon(q^*_\varepsilon)\}$ be the set of realizations where buyers are unconstrained, and $\mathcal{C} = \mathcal{U}^c_t(h^t)$ the set where they are constrained.

Suppose there is a monetary equilibrium. In this economy, it is less natural to transform the choice of $\hat{m}$ into a choice of q, since q is a function of the realized ε. Hence, we write the problem in terms of the choice of $L = \phi_{+1}\hat{m} + D$. The usual manipulations allow us to write the objective function as

$$J_t(L) = -\frac{i}{\alpha}L + \int\limits_{\mathcal{U}_t(h^t)} \left[u_\varepsilon\left(q_\varepsilon^*\right) - f_\varepsilon\left(q_\varepsilon^*\right)\right] dF_t\left(\varepsilon|h^t\right) + \int\limits_{\mathcal{C}_t(h^t)} \left[u_\varepsilon \circ g_\varepsilon(L) - L\right] dF_t\left(\varepsilon|h^t\right).$$

Observe that this problem, and hence the solution L_t, depend on i/α and of the distribution of shocks, but not on the debt limit D. Exactly as in the baseline model, $L_t = \phi_{+1}\hat{m} + D$ is pinned down, and any exogenous changes in D are neutralized by endogenous changes in real balances, as long as money is valued.

Hence, having purchases vary by size does not imply credit is essential in monetary equilibrium. Although we do not need it for the above argued, for the record, the equilibrium L_t solves

$$\int\limits_{\mathcal{C}_t(h^t)} \left[\frac{u_\varepsilon'\left(q_{\varepsilon,i}\right)}{f_\varepsilon'\left(q_{\varepsilon,i}\right)} - 1\right] dF_t\left(\varepsilon|h^t\right) = \frac{i}{\alpha},$$

and intuitively straightforward generalization of (13).

6.2 Real Assets

Consider replacing currency by an interest-bearing asset a – say, a Lucas tree in fixed supply A, with a dividend $\delta > 0$ that arrives every period in the CM. If the price of a is ψ, the CM constraint becomes $x = \ell + \delta a + \psi(a - \hat{a}) - d$, and the DM constraint becomes $f(q) \leq D + (\psi + \delta)\hat{a}$. There is no analog to Lemma 2 saying the latter constraint must bind. In fact, as is well known, it binds iff $A\gamma$ is low (e.g., see Lester et al. 2012). If it does not bind then $\psi = \psi^* = \gamma/r$ and $q = q_0$. To avoid a technicality (see Lester et al. 2012), assume $q_0 = q^*$, as must be the case with Walrasian pricing or Kalai bargaining.

Now assume γ is low enough for the DM constraint to bind. Then $\psi > \psi^*$, and the usual manipulations allow us to reduce the CM problem to

$$J(q) = \max_q \left\{u(q) - \left[1 + \frac{r\psi - \gamma}{\alpha(\psi + \gamma)}\right] f(q)\right\}.$$

The FOC is

$$u'(q) = \left[1 + \frac{r\psi - \gamma}{\alpha(\psi + \gamma)}\right] f'(q). \tag{23}$$

By the analog to Proposition 5, the solution q is generically unique and strictly decreasing in ψ, with $q \to q^*$ as $\psi \to \psi^*$ and $q \to 0$ as $\psi \to \infty$. This can be interpreted as a demand curve. Similarly, $f(q) = D + (\psi + \gamma) A$ can be interpreted as a supply curve, and it is strictly increasing, with $q \to g[D + (1+r)\psi^* A]$ as $\psi \to \psi^*$.

Hence, if $g[D + (1+r)\psi^* A] < q^*$ there is a unique equilibrium with $\psi \in (\psi^*, \infty)$ and $q \in (0, q^*)$, and if $g[D + (1+r)\psi^* A] \geq q^*$ there is a unique equilibrium with $\psi = \psi^*$ and $q = q^*$. The former case, where liquidity is scarce and the asset bears a premium, obtains iff $D + (1+r)\psi^* A < f(q^*)$. In this case increases in D shift supply but not demand, increasing q and decreasing ψ. So credit is essential and conditions are not neutral in this model. The reason is that a is not a fiat asset. When $\gamma = 0$ we recover the model with fiat money, where (23) says demand is perfectly inelastic, in which case changes in supply coming from changes in D do not affect q. With fiat money, an increase in D lowers the value of money ϕ, but $D + \phi M$ and hence q stay the same. This cannot happen with a real asset, since when the price ψ falls demand increases because the asset in addition to providing liquidity also is a better store of value – the dividend-price ratio is higher.

Since it is interesting, we also show how to endogenize the debt limit D with a real asset. The analog to (21) is[18]

[18] This is derived from the autarky payoff, $r\underline{W}(0,0) = (1+r)U_0$, and the CM payoff with a real asset,

$$rW(0,0) = (U_0 - \psi\hat{a})(1+r) + (\psi + \delta)\hat{a} + \alpha[u(q) - f(q)].$$

$$\Phi(D) = \begin{cases} \frac{\alpha\delta}{r} J \circ q(D) + \frac{\delta}{r} \frac{r\psi(D) - \gamma}{\psi(D) + \gamma} D & \text{if } D < f(q^*) - \frac{1+r}{r}\gamma \\ \frac{\alpha\delta}{r} [u(q^*) - f(q^*)] & \text{if } D \geq f(q^*) - \frac{1+r}{r}\gamma \end{cases}$$

where it is understood in the first branch that $q(D)$ and $\psi(D)$ come from the intersection of the supply and demand functions discussed above, which generally depend on D. Notice $\Phi(D)$ now only has two branches, while it had three with fiat money. The reason is simple: with fiat money, M is not valued in the middle branch, where D is enough to get at least q_i. But with a real asset, A can never be not valued, so the only distinction is where A is big enough to get q^*. Still, a fixed point $\hat{D} = \Phi(\hat{D})$ is an endogenous debt limit.

This extension does not overturn the main result, that D is irrelevant in monetary equilibrium, because the above analysis does not have money. So, we bring M back into the economy and try to construct a monetary equilibrium. With $f(q) = D + \phi_{+1}\hat{m} + (\psi + \gamma)\hat{a}$, the CM problem can be reduced to

$$\begin{aligned} J(\hat{m}, \hat{a}) &= \max_{\hat{m},\hat{a}} \left\{ u \circ g \left[\phi_+ \hat{m} + D + (\psi + \gamma)\hat{a} \right] - \phi_+ \hat{m} - D - (\psi + \gamma)\hat{a} \right\} \\ &\quad + \frac{1}{\alpha}(\psi + \gamma)\hat{a} - \frac{1+r}{\alpha}\psi\hat{a} + \frac{1}{\alpha}\phi_{+1}\hat{m} - \frac{1+r}{\alpha}\phi\hat{m} \end{aligned}$$

If both money and asset are used, the FOCs are

$$\begin{aligned} \frac{u'(q)}{f'(q)} - 1 &= \frac{i}{\alpha} \\ \frac{u'(q)}{f'(q)} - 1 &= \frac{\psi r - \gamma}{\alpha(\psi + \gamma)} \end{aligned}$$

which implies that

$$q = q_i, \; \psi = \bar{\psi} = \frac{(1+i)\gamma}{r - i}$$

and the equilibrium exists iff $q_i > q(D)$, where $q(D)$ is solved from the supply and demand functions discussed above. Note that in such an equilibrium, the price of asset is constant, and money can coexist with asset only if there is a deflation.

As in the baseline model, if we change D, $\phi M = f(q_i) - D - (\bar{\psi} + \gamma) A$ adjusts to keep $q = q_i$ the same as long as we stay in monetary equilibrium. So changes in D are neutral, in the sense that they cannot affect $q = q_i$.

6.3 Breaking Neutrality

Consider the model with shocks, but now assume they are realized ex ante shocks, before buyers choose $\hat{m} = \hat{m}_\varepsilon$. The choice of $\hat{m}_\varepsilon$ maximizes

$$J_\varepsilon = -i\phi_{+1}\hat{m}_\varepsilon + \alpha\left[u_\varepsilon(q) - \phi_{+1}\hat{m}_\varepsilon - D\right].$$

Let $q_{\varepsilon,i}$ solve

$$\frac{u'_\varepsilon(q_{\varepsilon,i})}{f'_\varepsilon(q_{\varepsilon,i})} - 1 = \frac{i}{\alpha}. \tag{24}$$

If $D \geq f_\varepsilon(q^*_\varepsilon)$ the buyer does not use money and $q = q^*_\varepsilon$. If $f_\varepsilon(q_{\varepsilon,i}) \leq D < f_\varepsilon(q^*_\varepsilon)$ the buyer is constrained by his debt limit, so $q < q^*_\varepsilon$, but he still does not use money. If $D < f_\varepsilon(q_{\varepsilon,i})$ the buyer chooses $\hat{m}_\varepsilon > 0$ and consumes $q_{\varepsilon,i}$.

Let Γ_1 be the set of buyers with low ε that consume q^*_ε, Γ_2 the set with intermediate ε that are constrained but do not use money, and Γ_3 the set with high ε who use both money and credit in the DM. When D increases, Γ_1 expands as more buyers are unconstrained, while Γ_3 shrinks as fewer buyers use money. For the buyers moving from one set to another and those staying in Γ_2, q strictly increases with D; for those staying in Γ_1 and Γ_3, q remains unchanged. Hence credit is not inessential, and changes in D are not neutral, because changes in credit conditions affects agents differently when they are heterogeneous with respect to money holdings.

Proposition 9 *With ex post ε shocks, in monetary equilibrium credit is not essential and changes in credit conditions are neutral. With ex ante shocks credit is essential and consumption q increases with D.*

Now suppose monitoring is heterogeneous, in the sense that δ is random across DM meetings. In any DM meeting there is a repayment constraint, as in the baseline model, implying a debt limit that is increasing in δ. There is thus a distribution of D resulting from random δ described by, say, $F(D)$ on $\left[\underline{D}, \overline{D}\right]$. Given $\hat{m}$ and the realized D, $q = q^*$ if $\phi m + D \geq f(q^*)$ and $q = g(\phi m + D)$ otherwise. It is easy to see that there is a unique $\widehat{D} = f(q^*) - \phi \hat{m}$ below which the buyer is constrained and above which he is not.

A buyers' objective function is

$$\begin{aligned} J &= -\frac{i}{\alpha}\phi_{+1}\hat{m} + \int_{\widehat{D}}^{\overline{D}} \left[u(q) - f(q^*)\right] dF(D) \\ &\quad + \int_{\underline{D}}^{\widehat{D}} \left[u \circ g\left(\phi_{+1}\hat{m} + D\right) - \left(\phi_{+1}\hat{m} + D\right)\right] dF(D) \end{aligned}$$

The FOC for $\hat{m} > 0$ is

$$\int_{\underline{D}}^{g(q^*)-\phi_{+1}\hat{m}} \left[\frac{u' \circ g\left(\phi_{+1}\hat{m} + D\right)}{f' \circ g\left(\phi_{+1}\hat{m} + D\right)} - 1\right] dF(D) = \frac{i}{\alpha}. \tag{25}$$

The DM allocation is random, and its distribution depends on the distribution of D. Again credit conditions affect the allocation, and credit is essential. And in monetary equilibrium money is also essential, as it it increases q in meetings where buyers are constrained.[19]

Proposition 10 *With random monitoring, credit is essential and credit conditions matter in monetary equilibrium.*

[19] Although credit and money can both matter with random ρ, the welfare implications are ambiguous: the allocations with money and credit and with only money cannot be Pareto ranked, in general. In fact, even using ex ante expected utility as a welfare measure, examples show that an economy with both money and credit can be better or worse off than one with only monetary.

7 Conclusion

This paper studied several models designed to help us understand the relationship between money and credit. We were primarily interested in asking if it is useful to have both when money and credit are substitutes as instruments of intertemporal exchange. We considered both exogenous and endogenous limits on debt and deflation, and considered a general class of mechanisms for determining the terms of trade. We allowed imperfect monitoring, which is important for the reasons discussed in Wallace (2013): "If we want both monetary trade and credit in the same model, we need something between perfect monitoring and no monitoring. As in other areas of economics ... extreme versions are both easy to describe and easy to analyze. The challenge is to specify and analyze intermediate situations." We think it is natural and tractable to assume random monitoring.

To summarize the results, we discussed the existence of equilibria where money and/or credit are used. We know monetary equilibrium exists at least for monetary policies that are not too loose (i.e., inflation is not too high) iff the debt limit D is low, and when money is used it is essential. We also showed the (stationary) monetary equilibrium is generically unique when it exists, although this was not critical at all to the main result. The main result is that whenever money is used, credit is not essential and changes in debt limits are neutral, at least in the baseline model. With certain types of heterogeneity, however, this result can be overturned. We still think the result is interesting. Consider Ricardian equivalence in macro, Modigliani-Miller irrelevancy in finance, or Kareken-Wallace exchange rate indeterminacy in international. One can always try to find loopholes and overturn such results, but there is still an element of truth. We would like to think this is also the case for our results on credit.

Appendix

This Appendix provides proofs of results that are not obvious from the discussion in the text, and sketches the results for the model with endogenous debt and tax limits when punishment involves allowing deviators to continue in the DM but only using cash.

Proof of Lemma 1 and its Corollaries: Consider buyers, for whom (3)-(4) hold at equality in monetary equilibrium. Also, as established in the text, buyers are constrained and hence $q < q^*$ in any monetary equilibrium. Differentiate these and the budget equation to get

$$\begin{bmatrix} -wU_{11}^b + U_{21}^b & -wU_{12}^b + U_{22}^b & 0 \\ -\phi U_{11}^b & -\phi U_{12}^b & \phi_{+1}\beta V_b'' \\ 1 & w & \phi \end{bmatrix} \begin{bmatrix} dx \\ d\ell \\ d\hat{m}_b \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ dA \end{bmatrix}$$

where V_b'' is well defined from (**??**) and the assumption $g \in \mathcal{C}^2$ on $(0, q^*)$. The determinant of the square matrix is $\Delta_1 = \phi_{+1}\beta V_b'' \left(w^2 U_{11}^b - 2wU_{21}^b + U_{22}^b\right) + \phi \left|U^b\right| > 0$ by the SOC, which must hold in equilibrium. Then

$$\frac{d\hat{m}_b}{dA} = \Delta_1^{-1}\phi \left|U^b\right| = 0,$$

by Assumption 1. Hence, $\hat{m}_b$ is independent of A. By (4), so is $U_1^b(\cdot)$, say $U_1^b(\cdot) = \Lambda_b$. By (3), so is $U_2^b(\cdot) = \Lambda_b w$. By the envelope theorem, $W_b'(\cdot) = \Lambda_b$.

That takes care of buyers in a monetary equilibrium. In a nonmonetary equilibrium, for buyers, we have

$$\frac{dU_1^b}{dA} = -\Delta_0^{-1} \left|U^b\right| = 0$$

where $\Delta_0 = -\left(w^2 U_{11}^b - 2wU_{21}^b + U_{22}^b\right) > 0$. Again, $U_1^b(\cdot) = \Lambda_b$ etc. This completes the argument for buyers. The argument for sellers is similar. ■

Proof of Lemma ??: Suppose $L \geq p^*$. Then $V_b'(\cdot) = W_b'(\cdot) = 1$, because the terms of trade $(p, q) = (p^*, q^*)$ are independent of L when the constraint is slack.

By the FOC for $\hat{m}$ at equality, $\phi = \beta\phi_{+1}$. Since $\phi/\phi_{+1} = 1 + \mu$, this contradicts $\mu > \beta - 1$. In the limiting case of the Friedman rule, $\mu = \beta - 1$, money can be held even if the constraint is slack, but in this case money does not accomplish anything – payoffs would be the same if $M = 0$.

Proof of Lemma 5: In the text, (11) reduced the buyer's problem to $\max_q J(q;i)$ st $q \in [0, \bar{q}]$, where $J(q;i) = u(q) - (1 + i/\alpha) f(q)$ is $\mathcal{C}^2$ and $J(0;i) = 0$. Clearly there exists a solution since J is continuous. By Assumption 3, $q = 0$ is not a solution, because there exists $q > 0$ such that $J(q,0) > 0$. Hence for i not too big there exists a $q_i > 0$ maximizing $J(q;i)$. At any such q_i, the FOC $J_q(q_i;i) = 0$ holds, although there might be multiple local maximizers, as shown in Figure 5. The higher curve is $J(q;i)$ for $i = i_1$ and the lower curve is $J(q;i_2)$ for $i_2 > i_1$.

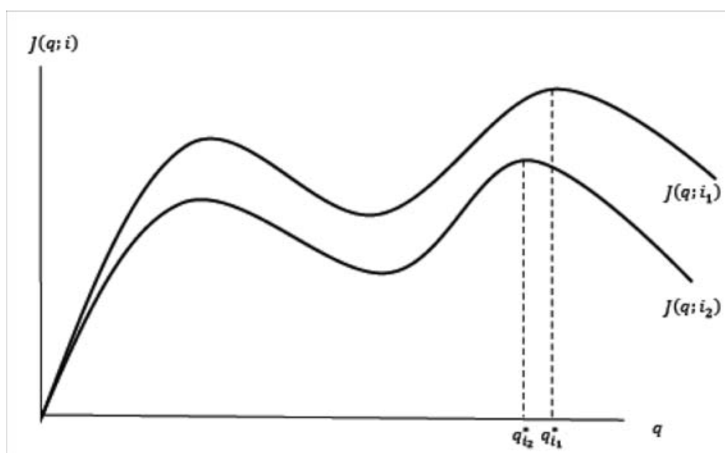

Figure 5: Uniqueness and Monotonicity

At any local maximum $J_q(q;i) = u' - (1 + i/\alpha) f' = 0$ and $J_{qq} = u'' - (1 + i/\alpha) f'' < 0$. We claim the global maximizer is unique for generic i. To see this suppose $J(q_1^*;i) = J(q_2^*;i) = \max_q J(q;i)$ with $q_2^* > q_1^*$. Increase i to $i + \varepsilon$. Since $J_i(q;i) = -f(q) < 0$ and $f(q)$ is increasing, $J_i(q_2^*;i) < J_i(q_1^*;i)$. Thus $J(q_1^*;i+\varepsilon) > J(q_2^*;i+\varepsilon)$, and now the global maximizer is unique at some q

close to q_1^*.

Increasing i_1 to i_2, J shifts down, as shown. Each local maximizer shifts to the left, $\partial q/\partial i = f'/[u'' - (1+i/\alpha) f''] < 0$. In particular, the global maximizer shifts to the left, q_i is a decreasing function of i when it is single valued. If we continue to increase i, we might reach a nongeneric point $\widetilde{i}$ where there are multiple global maximizers, say q_1^* and $q_2^* > q_1^*$. By the argument used above, for $\widetilde{i}+\varepsilon$ the unique global maximizer is close to q_1^* and for $\widetilde{i}-\varepsilon$ the unique global maximizer is close to q_2^*. So demand is a continuously decreasing and single-valued function except possibly for i in a set of measure 0, where demand is multiple valued and jumps to the left as i increases. Since for generic i there is a unique q_i, there cannot be multiple equilibria. ■

Proof of Proposition 3. First consider $L < p^*$. By A3 and the definition of p^*, we have $q = G_q(L) < q^*$. We prove $p = L$ by contradiction. Suppose $p \neq L$. We cannot have $p > L$, by A1, so $p < L$. Consider $p' = p+\delta < L$ and $q' = q+\epsilon < q^*$, which is feasible for small (δ, ϵ). If $\delta = [u(q') - u(q)]/\Lambda_b$, one can easily check that the buyer's surplus S_b does not change, while for the seller

$$dS_s = p' - v(q') - p + v(q) = u(q') - v(q') - u(q) + v(q). \tag{26}$$

Since $q^* > q' > q$, $u(q) - v(q)$ is increasing in q. Therefore S_s increases, contradicting A4.

Next, consider $L \geq p^*$. We prove $q = q^*$ by contradiction. Suppose $q = G_q(L) < q^*$. We know $p = P(L) < L^*$ by A3. Let $p' = p + \delta$ and $q' = q + \epsilon$. As in the previous step, one can check (p', q') dominates (p, q), contradicting A4. Suppose instead $q > q^*$. Let $p' = p - \delta$ and $q' = q - \epsilon > q^*$, where $\delta = u(q) - u(q')$, which satisfies A1 and A3 for small (δ, ϵ). One can check that S_b does not change while the change in S_s is the same as (26). Since $q > q' > q^*$, $u(q) - c(q)$ is decreasing in q. Therefore S_s increases, contradicting A4. Hence, $L \geq p^*$ implies $q = q^*$, which implies $p = p^*$ by the definition of p^* and A3. Hence, $g(p^*) = q^*$.

By A3, g is strictly increasing. By A2, $g(0) = 0$. By definition of p^*, $g(p^*) = q^*$. This completes the proof. ■

Proof of Proposition 4. Figure 6 shows $u(q)$ and $(1 + i/\alpha) v(q)$. Pick p^o such that $(1 + i/\alpha) v(q^o) < p^o < u(q^o)$, which is possible given $q^o \in [0, \hat{q}]$. Draw a line through (q^o, p^o) with slope $u'(q^o)$, labelled in the graph $(1 + i/\alpha) f^o(q)$. Now define $f(q)$ on $[0, \bar{q}]$ by first rotating $(1 + i/\alpha) f^o(q)$ to get $f^o(q)$, then truncating it above by $u(q)$ and below by $v(q)$. Since $f(\cdot)$ is strictly increasing, $g(\cdot) = f^{-1}(\cdot)$ is well defined, and a trading mechanism is given by (9). This mechanism is consistent with trading q^o and $p^o = f(q^o)$ ex post, in the DM, because $v(q^o) \leq p^o \leq u(q^o)$. And it is consistent with the ex ante decision to bring enough liquidity out of the CM, because q^o is the global maximizer of a buyer's objective function $J(q; i) = u(q) - (1 + i/\alpha) f(q)$. So we can support $q^o \leq q^*$.

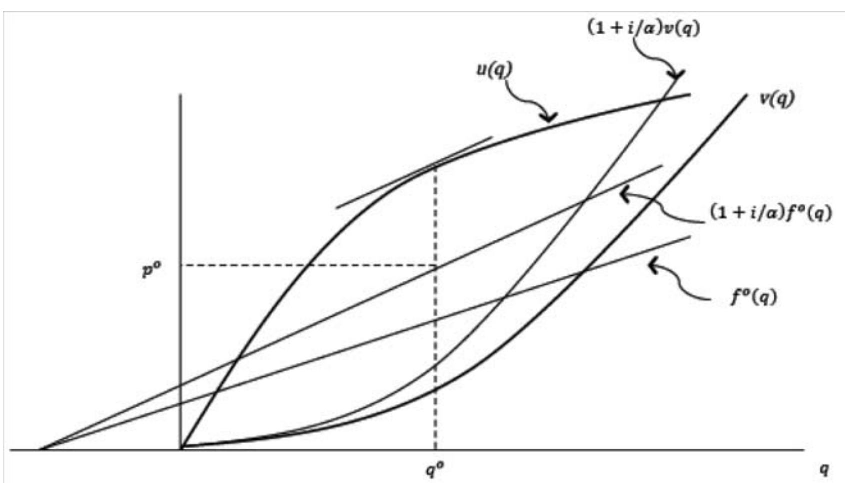

Figure 6: The HKW Mechanism

Now consider $q^o > q^*$. We claim that such a q^o cannot be supported by a

Pareto efficient mechanism. Although the buyer has the ex ante incentive to take enough liquidity out of the CM to pay p^o and get q^o, ex post in a DM meeting there is an alternative (p, q) that Pareto dominates (p^o, q^o), involving a reduction in q from q^o toward q^* combined with a reduction in p^o. Given A4, we cannot support $q^o > q^*$. Note that this is not an issue for $q^o \leq q^*$, since when a buyer only brings enough to get q^o, renegotiation towards $q^* > q^o$ violates A1. So we can construct mechanisms that deliver any $q^o \leq q^*$ in the case with $\hat{q} > q^*$. If $\hat{q} < q^*$ we cannot support q^*, and $\hat{q}$ is the highest q that is incentive feasible. ■

Alternative punishment: Suppose now that if an agent is caught reneging, he is banned from using credit in the DM, but can continue using cash. The punishment payoff is

$$\underline{W}(\phi m) = \max_{x,\ell,\hat{m},q} \left\{U^b(x,\ell) + \beta\alpha[u(q) - f(q)] + \beta\underline{W}\left(\phi_+\hat{m}\right)\right\}$$
$$\text{st } \phi m + w(1-\ell) = x + \phi\hat{m} \text{ and } f(q) \leq \phi_{+1}\hat{m},$$

In monetary equilibrium, the punishment payoff is reduced to

$$\underline{W}(\phi m) = \frac{1+r}{r}U_0 + \phi m - f(q_i)\frac{i}{r} + \frac{\alpha}{r}[u(q_i) - f(q_i)]$$

and in nonmonetary equilibrium $\underline{W}(\phi m)$ is the same as in the text. As always, the tax payment constraint is redundant in nonmonetary equilibrium. In monetary equilibrium, it can be reduced to $(r+\tau)T \leq \tau i D$.

Given an incentive compatible policy, the debt repayment constraint is again $d \leq \Phi(D)$, where now

$$\Phi(D) \equiv \begin{cases} \delta D + \delta\dfrac{i-r}{r}f(q_i) & \text{if } D < f(q_i) \\ \dfrac{\delta\alpha}{r}[u \circ g(D) - D] & \text{if } f(q_i) \leq D < f(q^*) \\ \dfrac{\delta\alpha}{r}[u(q^*) - f(q^*)] & \text{if } f(q^*) \leq D \end{cases}$$

if we select the monetary equilibrium when it exists. In monetary equilibrium, the fixed point is

$$D = \frac{\delta}{1-\delta} \frac{i-r}{r} f\left(q_i\right), \tag{27}$$

which satisfies $0 \leq D < f\left(q_i\right)$ iff $r \leq i < r/\delta$. Substitute this into the constraint $(r+\tau) T \leq \tau i D$ to get

$$\frac{r-i}{r} f\left(q_i\right) \leq 0. \tag{28}$$

Therefore $r \leq i < r/\delta$ is necessary and sufficient condition for a monetary equilibrium, and no deflationary monetary policy is feasible.

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