A Mechanism Design Approach to Financial Frictions*

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Abstract

We use a mechanism design approach to illustrate how different environments of private information and limited commitment generate different financial frictions. We show that limitations in commitment and private information lead to a permanent income theory of consumption with borrowing constraints.

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1 Introduction

In a thought provoking article "Can a "Credit Crunch" Be Efficient?" Edward Green and Soo Nam Oh use a mechanism design approach to present a model of financial intermediation in which phenomena qualitatively resembling a "credit crunch" occur but are efficient. In this short paper, we extend and modify the model of Green and Oh in order to examine how different environments of private information and limited commitment generate different financial frictions. Following a tradition of mechanism design, which considers the market structure as an equilibrium outcome of the underlying environment, we ask questions such as: Which markets are open? Which contracts are used? Which institutions arise? We find that the model of Green and Oh is a useful benchmark to explain the recent literature on the mechanism design approach to financial frictions.

2 Model of Green and Oh

There is a continuum of agents with population size being normalized to one. All agents live for three dates, dates 0, 1, and 2. There is one homogeneous goods at each date. At date 0, everyone is identical and signs a contract. At date 1, an idiosyncratic income shock realizes, and each agent receives either a high income (or endowment) y_h with probability π_h, or receives a low income y_l with probability π_l, where $0 < y_l < y_h$ and $\pi_h + \pi_l = 1$. We assume that exactly a fraction π_h of agents receives the high income and that a fraction π_l receives the low income so that there is no uncertainty about the aggregate income $\overline{y} = \pi_h y_h + \pi_l y_l$ at date 1. At date 2, everyone receives an identical income of z, where

$$0 < y_l < z < \overline{y} < y_h.$$

The agent enjoys utility of consuming goods at dates 1 and 2, and his or her preferences at date 0 are determined by the expected utility

$$EU = \sum_{i=h,l} \pi_i \left[u(c_{1i}) + \beta u(c_{2i}) \right], \tag{1}$$

where c_{ti} denotes date t consumption of the agent whose date 1 income is y_i $(i = h, l)$, and $\beta \in (0, 1)$ denotes the common discount factor. We assume

$u(c)$ satisfies the usual regularity conditions: $u'(c) > 0, u"(c) < 0, u'(0) = \infty$ and $u'(\infty) = 0$. Agents can store goods which allow them to transform x units of goods stored at date 1 into $(1/\beta) \cdot x$ units of goods at date 2.

2.1 Public Information and Full Commitment

Before introducing the financial frictions, let us examine the economy in which all the information about individual income and storage is public information and individuals can fully commit to obey the contract (i.e., there is no limitation on enforcing contracts). Then the competitive economy corresponds to a solution of the planner's problem in which consumption and the storage $(c_{1h}, c_{2h}, c_{1l}, c_{2l}, x)$ are chosen to maximize the expected utility of a typical agent (1) subject to the resource constraints:

$$\begin{aligned} \pi_h c_{1h} + \pi_l c_{1l} &= \overline{y} - x, \\ \pi_h c_{2h} + \pi_l c_{2l} &= z + \frac{1}{\beta} x. \end{aligned}$$

When the storage is non-negative, the resource constraints can be combined as

$$\pi_h(c_{1h} + \beta c_{2h}) + \pi_l(c_{1l} + \beta c_{2l}) = \overline{y} + \beta z, \tag{2}$$

and the solution is

$$\begin{aligned} c_{1h} &= c_{1l} = c_{2h} = c_{2l} = \frac{1}{1+\beta}\left(\overline{y} + \beta z\right), \\ x &= \frac{\beta}{1+\beta}\left(\overline{y} - z\right) > 0. \end{aligned}$$

Thus, if there is no friction of information and commitment, individual consumption does not depend upon idiosyncratic income shock, and depends only upon aggregate income, because agents can perfectly insure against the idiosyncratic income risks. With perfect risk sharing, the marginal utility of consumption is equalized across agents. When all agents have identical preferences, consumption is equalized too. Moreover, everyone's consumption should be smoothed over time through storage (whose rate of return is equal to the time preference rate $1/\beta$).

The solution of this planner's problem can be considered as an outcome of a competitive economy in which intermediaries compete with each other to offer contracts for state-contingent net transfers. Under Bertrand-style competition between intermediaries, the contract offered by all intermediaries in equilibrium would be the contract which maximizes the expected utility of the customers subject to the resource constraints. This frictionless competitive economy - so called Arrow-Debreu economy - serves as a benchmark. However, the prediction appears to contradict the observation that individual consumption significantly depends upon the individual's income (in addition to the aggregate income per capita) in household panel data. (See Altug and Miller (1990), Cochrane (1991) and Mace (1991) for example). The next sections depart from such a frictionless economy in order to explain the household data.

2.2 Private Information of Individual Income

Green and Oh (1991) (which is based upon a classic paper of Green (1987)) considers private information about individual income as a key friction to explain phenomena like "credit." If individual income is private information, the perfect risk sharing achieved in the Arrow-Debreu economy is no longer compatible with the incentive constraints since everyone would claim to have earned the low income to receive a positive transfer. Thus, when individual income is private information, the allocation rule has to satisfy an incentive constraint which requires high-income agents not to pretend to be a low-income type:

$$u(c_{1h}) + \beta u(c_{2h}) \geq u(y_h + c_{1l} - y_l) + \beta u(c_{2l}). \tag{3}$$

The left hand side (LHS) denotes the utility when the high-income agent tells the truth to the intermediary about her income. The right hand side (RHS) denotes instead the utility when the high-income agent misrepresents herself as a low-income agent. By pretending to be a low-income agent, she receives the transfer $c_{1l} - y_l$ and consumes $y_h + c_{1l} - y_l$ at date 1, and pays $z - c_l$ and consumes $z - (z - c_l) = c_l$ at date 2. To derive this, we have assumed that the storage was public information so that no one could use storage privately in order to adjust consumption across dates.

Thus, in this environment, the optimal contract $(c_{1h}, c_{2h}, c_{1l}, c_{2l})$ can be found by maximizing the expected utility subject to the resource constraint and the incentive constraint (3). Hence, the associated Lagrangian is

$$\begin{aligned}\mathcal{L} &= \pi_h\left[u(c_{1h})+\beta u(c_{2h})\right]+\pi_l\left[u(c_{1l})+\beta u(c_{2l})\right]\\ &\quad +\lambda\left[\overline{y}+\beta z-\pi_h(c_{1h}+\beta c_{2h})-\pi_l(c_{1l}+\beta c_{2l})\right]\\ &\quad +\mu_h\left[u(c_{1h})+\beta u(c_{2h})-u(y_h+c_{1l}-y_l)-\beta u(c_{2l})\right]\end{aligned}$$

where λ and μ_h are the Lagrange multipliers associated to the resource constraint and to the incentive constraint. The first order conditions of such a problem require

$$\left(1+\frac{\mu_h}{\pi_h}\right)u'(c_{1h})=\lambda=\left(1+\frac{\mu_h}{\pi_h}\right)u'(c_{2h})$$

$$u'(c_{1l})-\frac{\mu_h}{\pi_l}u'(y_h+c_{1l}-y_l)=\lambda=\left(1-\frac{\mu_h}{\pi_l}\right)u'(c_{2l}).$$

Then, since $u'(y_h+c_{1l}-y_l)<u'(c_{1l})$, we obtain that

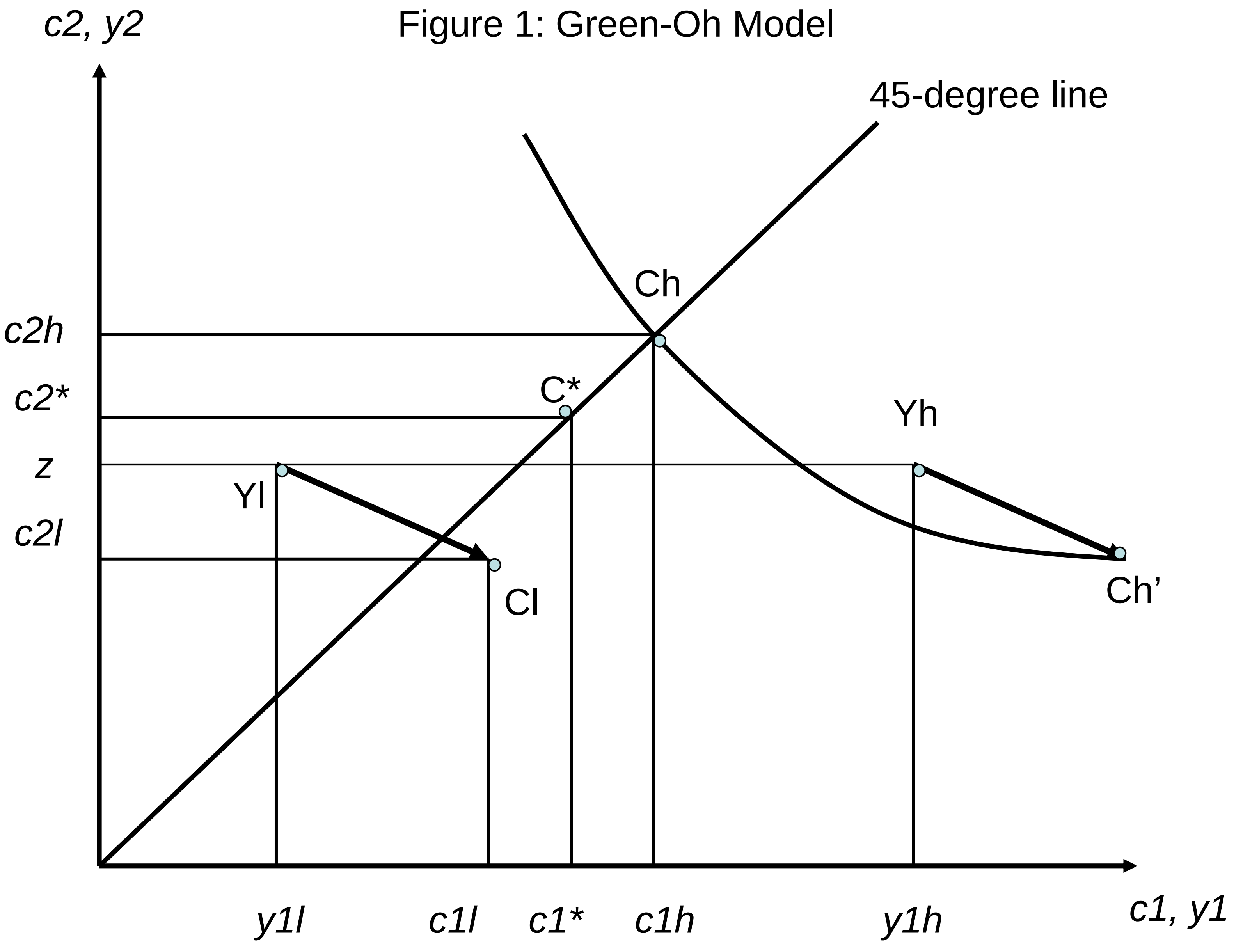
$$c_{2l}<c_{1l}<c_{1h}=c_{2h}.$$

In this environment, a "credit"-like arrangement arises as an endogenous outcome of the private information about the individual income. In order to prevent the high-income agent from pretending to be a low-income agent, the intermediary cannot transfer a positive amount to the low-income agent in both periods. Those who consume more than their income at present (like borrowers) have to pay by consuming less than their income in future. Thus, agents cannot be fully insured, since low-income agents consume less than high-income agents.

Moreover, although consumption is smooth over time for the high-income agent, consumption of the low income agent is skewed towards date 1. These are because skewing the consumption of low-income agents towards date 1 reduces the gains of the high-income agent from misrepresenting herself as a low-income agent, and in turn, relaxes the incentive constraint.

Figure 1 illustrates the allocation when individual income is private information. The horizontal axis measures the date 1 income and consumption, while the vertical axis measures those of date 2. Under public information and full commitment, the competitive economy achieves the first best allocation in which consumption is equalized across agents and across time at point C^* in the figure. When the individual income is private information,

Figure 1: Green-Oh Model



the consumption of the low income agent is at point C_l, in which consumption is larger than income at date 1 but it is smaller than income at date 2 (in order to repay the 'debt'). Moreover, consumption at date 1 is larger than consumption at date 2 ($c_{1l} > c_{2l}$) for the low-income agent. Consumption of the high-income agent is balanced at point C_h. Point C'_h is consumption if the high-income agent pretends to be a low income agent to receive the net transfer

$$C'_h = Y_h + (C_l - Y_l),$$

where capital letters denote vectors such as $C_l = (c_{1l}, c_{2l})$. The point C'_h and point C_h are on the same indifference curve as the incentive constraint is binding for the high income agent.

When we compare the present value of consumption and income, we learn that there is a transfer from the high-income agents to the low-income agents. Both consumption allocations C_h and C_l are closer to the first best allocation C^* than incomes Y_h and Y_l in terms of the present value. But consumption is not equalized across agents, because insurance must be imperfect with private information.

3 Private Information of Individual Income and Storage (Cole-Kocherlakota)

Cole and Kocherlakota (2001) consider an environment in which not only individual income but also storage (saving) are private information. Then the high-income agent who pretends to be a low-income agent can use storage privately in order to smooth consumption. Let $V(W)$ be date 1 value function, when the agent chooses consumption and storage in order to maximize the utility subject to the constraint that the present value of consumption is equal to W, i.e.,

$$\begin{aligned} V(W) &= \underset{c_1, c_2}{Max} \left[u(c_1) + \beta u(c_2) \right], \\ subject\ to\ W &= c_1 + \beta c_2. \end{aligned}$$

By the envelope theorem, the value function satisfies $V'(W) = u'(c_1) = u'(c_2)$. The incentive constraint of the high income agent has to be modified to take into account the additional friction as

$$u(c_{1h}) + \beta u(c_{2h}) \geq V(y_h - y_l + c_{1l} + \beta c_{2l}). \tag{4}$$

The LHS denotes the utility of the high-income agent who tells the truth. In the RHS, the utility of the high-income agent who misrepresents to be a low income agent is the function of her wealth, which is the sum of the date 1 income gap (that the high-income agent hides) and of the present value of consumption of the low-income agent.

In this set-up, the optimal contract $(c_{1h}, c_{2h}, c_{1l}, c_{2l})$ maximizes the expected utility subject to the resource constraint and the incentive constraint (4). Using the Lagrangian

$$\begin{aligned}\mathcal{L} &= \pi_h \left[u(c_{1h}) + \beta u(c_{2h})\right] + \pi_l \left[u(c_{1l}) + \beta u(c_{2l})\right] \\ &\quad + \lambda \left[\overline{y} + \beta z - \pi_h(c_{1h} + \beta c_{2h}) - \pi_l(c_{1l} + \beta c_{2l})\right] \\ &\quad + \mu_h \left[u(c_{1h}) + \beta u(c_{2h}) - V(y_h - y_l + c_{1l} + \beta c_{2l})\right],\end{aligned}$$

the first order conditions can be arranged as

$$\begin{aligned}\left(1 + \frac{\mu_h}{\pi_h}\right) u'(c_{1h}) &= \lambda = \left(1 + \frac{\mu_h}{\pi_h}\right) u'(c_{2h}) \\ \lambda + \frac{\mu_h}{\pi_l} V'(y_h - y_l + c_{1l} + \beta c_{2l}) &= u'(c_{1l}) = u'(c_{2l}).\end{aligned}$$

Thus we have

$$\begin{aligned}c_{1h} &= c_{2h} = \frac{1}{1+\beta}\left(y_h + \beta z\right), \\ c_{1l} &= c_{2l} = \frac{1}{1+\beta}\left(y_l + \beta z\right).\end{aligned}$$

The present value of consumption of each agent is equal to the present value of his or her income here. There is no insurance nor transfer of wealth across agents. Because the high-income agent can announce the income which maximizes the present value of net transfer from the intermediary, the present value of net transfer must be zero for both high-income and low-income agents. The intermediary cannot cross subsidize agents, because all agents would choose to receive the subsidy and not to pay to the intermediary.

On the other hand, consumption is smooth over time. Therefore, Cole and Kocherlakota (2001) provides a mechanism design foundation of "permanent income theory of consumption" by Friedman (1967) - individual consumption reacts to idiosyncratic income shocks, even though agents can smooth consumption over time. This result is valuable because the optimal contract becomes simpler and more 'realistic,' when privation information about storage is added to a model with private information about income.

4 Limited Commitment

In a decentralized market economy, people often do not keep their promises and the intermediary cannot enforce contracts completely. This is a problem of limited commitment on the side of the agents and limited contract enforcement on the side of the intermediary. Suppose that the income is paid directly to the individual who cannot commit to pay a large fraction of income in future. It is not easy for the intermediary to enforce the individual to pay a large amount which contracts specify. In this setup the financial friction can arise endogenously. In fact, such limitations in commitment and enforcement can generate the financial friction even if individual income is public information.

To be more specific, suppose that the agent will not pay more than $\theta \in 0, 1)$ fraction of present and future income, and that nobody can take away any fraction of the individual's storage. Then consumption of low-income agent cannot be smaller than $1-\theta$ fraction of income at date 2:

$$c_{2l} \geq (1-\theta)z, \tag{5}$$

because the intermediary cannot force him to pay more than θz. At date 1, the high-income agent will not give up more than θ fraction of her wealth. The incentive constraint for a high-income agent to follow her intermediary's specified net transfer requires

$$u(c_{1h}) + \beta u(c_{2h}) \geq V((1-\theta)(y_h + \beta z)). \tag{6}$$

The optimal contract $(c_{1h}, c_{2h}, c_{1l}, c_{2l})$ maximizes the expected utility of a typical agent subject to the resource constraint and the two incentive constraints $(5, 6)$. If θ is sufficiently small, both incentive constraints are binding, and thus we have:

$$c_{1h} = c_{2h} = \frac{1}{1+\beta}(1-\theta)(y_h + \beta z)$$

$$c_{2l} = (1-\theta)z$$

$$c_{1l} = (1-\theta)y_l + \frac{\theta(\overline{y} + \beta z)}{\pi_l}. \tag{7}$$

The RHS of the first equation is permanent income of the high-income agent which no intermediary can take away—Holmstrom and Tirole (1999) call it "non-pledgeable" income. The second equation says consumption of the low-income agent is equal to his non-pledgeable income at date 2. In the RHS of the last equation, the first term is the non-pledgeable income of the low-income agent. The numerator of the second term is the fraction of the aggregate wealth which the intermediary can reallocate across agents—Holmstrom and Tirole call it "pledgeable" wealth.[1] Thus the optimal contract under such a severe limitation of contract enforcement requires the intermediary to allocate all the pledgeable wealth to the most needy agents, i.e., the low income agents at date 1. Here, unlike the previous examples of the private information economy, the low-income agent faces a binding contract enforcement constraint (loosely speaking, the bankruptcy constraint). A distinctive characteristic of an economy with limited commitment (but without private information) is that, although the resource base which contracts can reallocate is limited, there is no further restriction on how contracts can reallocate the pledgeable wealth.

5 Limited Commitment and Private Information

What happens if the individual income and storage are private information and the individual cannot commit to pay a large amount to the intermediary in future? Formally, the optimal contract $(c_{1h}, c_{2h}, c_{1l}, c_{2l})$ would maximize

[1] Kiyotaki and Moore (1997) consider an economy in which, instead of a fraction of future income, fixed assets such as real estates become the pledgeable wealth (collateral), exploring the interaction between the collateral value and aggregate production.

the expected utility subject to the resource constraint and the three incentive constraints $(4, 5, 6)$. As argued above, with the private information about individual income and storage, the intermediary cannot cross subsidize agents, and the present value of net transfer must be zero for all agents. Thus (6) is not binding. Then, using the Lagrangian

$$\begin{aligned} \mathcal{L} &= \pi_h [u(c_{1h}) + \beta u(c_{2h})] + \pi_l [u(c_{1l}) + \beta u(c_{2l})] \\ &+ \lambda [\overline{y} + \beta z - \pi_h(c_{1h} + \beta c_{2h}) - \pi_l(c_{1l} + \beta c_{2l})] \\ &+ \mu_h [u(c_{1h}) + \beta u(c_{2h}) - V(y_h - y_l + c_{1l} + \beta c_{2l})] \\ &+ \mu_l [c_{2l} - (1-\theta)z], \end{aligned}$$

the first order conditions can be arranged as

$$\begin{aligned} \left(1 + \frac{\mu_h}{\pi_h}\right) u'(c_{1h}) &= \lambda = \left(1 + \frac{\mu_h}{\pi_h}\right) u'(c_{2h}) \\ u'(c_{1l}) &= \lambda + \frac{\mu_h}{\pi_l} V'(y_h - y_l + c_{1l} + \beta c_{2l}) \\ u'(c_{2l}) &= \lambda - \frac{\mu_l}{\beta \pi_l} + \frac{\mu_h}{\pi_l} V'(y_h - y_l + c_{1l} + \beta c_{2l}). \end{aligned}$$

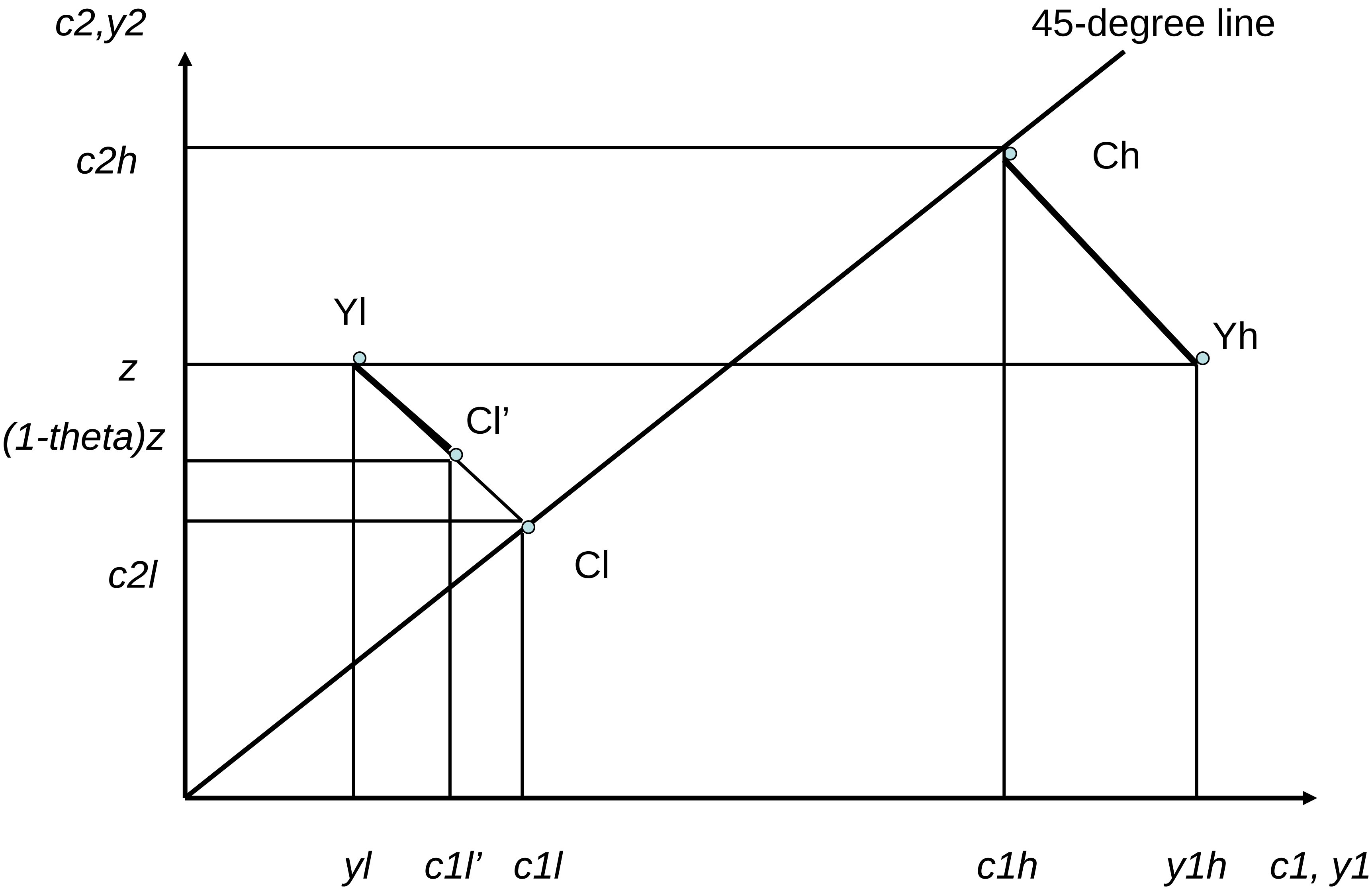
Then when θ is small enough, we have

$$\begin{aligned} c_{1h} &= c_{2h} = \frac{1}{1+\beta} (y_h + \beta z), \\ c_{1l} &= y_l + \beta \theta z, \\ c_{2l} &= (1-\theta) z. \end{aligned} \tag{8}$$

Thus, as in permanent income theory, the present value of the individual consumption is equal to the individual income−there is no insurance. Moreover, the low-income agent faces a binding borrowing constraint at date 1. He can borrow only against θ fraction of pledgeable future income. His date 1 consumption is equal to his current income and the present value of his pledgeable future income in (8). Therefore limitations in commitment and private information of income and saving lead to a permanent income theory of consumption with borrowing constraints−arguably the most common contract we observe in data.

In Figure 2, the points Y_h and Y_l show income of the high-income agent and the low-income agent. If the individual income and storage are private

Figure 2: Hidden Storage and Limited Commitment



information, but the individual can commit to pay in future, then the consumption of the high-income agent is $C_h = (c_{1h}, c_{2h})$ on the 45-degree line and the present values of consumption and income are equal (the line $Y_h C_h$ has a slope of $-(1/\beta)$). The consumption of the low-income agent is $C_l = (c_{1l}, c_{2l})$ on the 45-degree line, and again the present values of consumption and income are the same. This is a simplified version of Cole and Kocherlakota (2001).

If, in addition to the private information of the individual income and storage, the individual cannot commit to pay more than θ fraction of future income, then the consumption of the low-income agent becomes $C_l' = (c_{1l}, (1-\theta)z)$. The low-income agent wants to borrow as much as $c_{1l} - y_l$, but can only borrow up to $c_{1l}' - y_l$ at date 1 because he can commit to pay only θz at date 2. The consumption of the high-income agent is unchanged at C_h. The high-income agent is not constrained in her borrowing, because she lends to the intermediary instead of borrowing at date 1.[2]

6 Concluding Remark

In this article, we illustrate how 'credit' like arrangements arise endogenously and take a particular form in response to both private information of individual income and storage, and limited commitment. So far we have ignored the problem of the incentive constraint of the intermediary. But, how do we know that the intermediaries are trustworthy? What happens if the intermediary has private information or limited commitment? There is some literature on this.[3] Given that financial intermediaries experienced significant financing constraints during the recent financial crisis, we expect that active research will take place in a near future which accounts for incentive constraints of the intermediary in the tradition of mechanism design and general equilibrium literature.

[2] See Townsend (1989) for example for the early literature on the related topic. Ljungqvist and Sargent (2004) explain recent developments of optimal contract literature in infinite horizon frameworks.

[3] There is a vast literature on incentive constraint of the financial intermediaries from the perspective of microeconomics of banking. See Freixas and Rochet (2008). For a more mechanism design and/or general equilibrium tradition, see Krasa and Villamil (1992), Holmstrom and Tirole (1997), Gertler and Karadi (2011), Gertler and Kiyotaki (2010) and Gertler, Kiyotaki and Queralto (2011) for example.

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