Bank Deposit Insurance: Implications for Portfolios and Equilibrium Valuation of Risky Assets

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Abstract

We consider a model in which a government insurer guarantees deposits at commercial banks. In the basic specification deposit insurance is financed via lump-sum taxes. Commercial banks respond to the risk-shifting opportunity by holding high levels of risky assets. Their doing so raises risky-asset prices. We also determine the equilibrium when deposit insurance is financed using deposit-based or risky-asset-based insurance premia that are not actuarially fair. In both cases, we find that (1) insured commercial banks and uninsured shadow banks coexist under subsidized deposit insurance (2) and the asset price distortion is eliminated when the total subsidy to unsuccessful banks equals the total penalty to successful banks.

In the absence of deposit insurance, bank depositors must monitor banks to ensure that they have enough cash available to execute transfers of funds when called upon to do so. Such monitoring can be expensive. Additionally, adverse shocks to the banking system can result in bank runs. In order to relieve depositors of monitoring costs and eliminate the risk of bank runs, many countries have instituted deposit insurance.[1] However, deposit insurance entails risks of its own: when losses are large enough to lead to bank failure the downside is transferred from bank owners, creditors and depositors to taxpayers. With bankers enjoying all gains on investment but bearing only part of investment losses, it appears that they have an incentive to take on more risk than they would otherwise. This is, of course, the familiar moral hazard problem that accompanies insurance programs in which the insurer cannot completely monitor the behavior of the insured party: the insured party has an incentive to take actions that increase the value of the insurance. The implications of moral hazard in the context of deposit insurance were pointed out in the classic paper of Kareken and Wallace (1978); see also Pyle (1984) and Kareken (1990).

One expects that the distortion of bankers' incentives increases banks' demands for risky assets, resulting in higher equilibrium asset prices and excessive allocation of resources to risky projects; see Suarez (1993).[2] However, this is not a foregone conclusion: insured banks exist in an environment in which agents can avoid costly deposit insurance by holding assets directly or by transferring them to institutions that do not participate in deposit insurance. It is therefore not clear that the incentives that induce regulatory arbitrage in commercial banking will necessarily affect equilibrium prices. It is also not clear

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[1] The International Association of Deposit Insurers (www.iadi.org) reports that 113 countries had some form of explicit deposit insurance as of January 31 2014; see Demirguc-Kunt, Kane and Laeven (2008) for a discussion of factors that influence the adoption and design of deposit insurance by countries. Demirguc-Kunt and Kane (2002) document cross-country differences in the performance of deposit insurance programs.

[2] The expectation that deposit insurance increases risky asset prices is based on the general principle that agency problems resulting in risk-shifting behavior by agents lead to overvaluation of risky assets; see Franklin Allen and Gary Gorton (1993) and Franklin Allen and Douglas Gale (2000) for similar asset pricing implications from agency problems between investors and portfolio managers.

the extent to which, or even whether, the distortion of asset prices induced by deposit insurance depends on how the insurance is financed.

In this paper we investigate the effects of deposit insurance in a general equilibrium setting under the assumptions that banking is competitive and that agents can either turn over assets to commercial banks in exchange for insured deposits or hold assets outside the commercial banking system, thus avoiding both the direct benefits and costs of deposit insurance.[3] This setting allows us to ascertain how deposit insurance affects asset prices (1) when agents can transfer assets to institutions that do not participate in the insurance program (2) and under different financing regimes, for arbitrary levels of insurance premia.

Analysts of deposit insurance focus on whether the insurance program is actuarially fair or not (meaning that the insurance premium paid by each bank at each date equals the expected insurance transfer associated with whatever portfolio the bank chooses at that date). Actuarial fairness implies absence of distortion, at least under risk-neutrality: if the insurer is assumed to adjust premia one-for-one in response to changes in expected insurance transfers, then risk-neutral bankers are indifferent as to the existence or nonexistence of deposit insurance.[4] In that case transfers to failed banks induce no distortion in portfolios or asset prices. However, actuarial fairness is a very strong condition: it is altogether unrealistic to assume that real-world deposit insurers can accurately measure changes in riskiness of individual banks' portfolios and are able and willing to adjust premia promptly in response to such changes.

In practice deposit insurance premia are generally charged in proportion to either deposits or holdings of risky assets, with the factor of proportionality the same for all banks and all dates, and the same over all portfolio choices. It follows that deposit insurance subsidizes banks that experience low asset

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[3]Our specification is the opposite of that of Kareken and Wallace (1978), who analyzed deposit insurance in a setting in which the banking sector consists of a single monopolistic bank.

[4]For example, Prescott (2002) showed that if deposit insurance is actuarially fair and the insurer can observe bank behavior, then suitably chosen premia can eliminate the moral hazard problem. In earlier work, Merton (1977) used option pricing theory to derive an analytic formula for the actuarially fair level of the insurance premium.

returns, and therefore have a high probability of failing, and penalizes banks that experience high returns. Existence of these subsidies and penalties implies that such deposit insurance is not actuarially fair. Depending on the factor of proportionality, deposit insurance may subsidize banks in the aggregate or penalize them, with lump-sum taxes or transfers making up the shortfall or overage in the insurance fund.

Suppose first that deposit insurance subsidizes commercial banks. As a result equilibrium asset prices are distorted, with the direction of the distortion depending on whether the premium is charged against deposits or risky asset holdings. If the subsidy is high enough—meaning that the insurance premium per dollar of deposits issued or risky assets held is low enough—all agents use commercial banks, so as to take advantage of the underpriced deposit insurance. In that case the shadow banking system does not exist. A more interesting case occurs when the subsidy is lower (although still positive). In equilibrium individual agents are indifferent between patronizing commercial banks and shadow banks. This occurs because agents operating shadow banks can trade to their advantage based on the distortion of asset prices induced by deposit insurance. Their expected gain from doing so exactly equals the benefit of deposit insurance to commercial banks.

It is obvious that shadow banks can coexist with commercial banks when deposit insurance does not subsidize commercial banks. The result just stated implies the more surprising fact that this coexistence may also occur under subsidized deposit insurance (depending on premium levels). The coexistence result, although a direct implication of free entry plus the fact that shadow banks can trade advantageously due to the price distortions implied by deposit insurance, has not been stated before, to our knowledge. Analysts missed this result because they mostly used partial equilibrium analyses, in contrast to the general equilibrium setting specified here.[5]

At the boundary between an aggregate subsidy or penalty, deposit insurance is revenue neutral. In that case, the revenue from the insurance premia

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[5]A recent paper that analyzes the related problem of capital requirements in a general equilibrium setting is Harris, Opp and Opp (2014).

collected is equal to the insurance payments to failed banks, implying that lump-sum taxes and transfers are both zero. As far as we know, we are the first to study revenue-neutral deposit insurance in the academic literature. Our focus on revenue neutrality is motivated by the fact that, following public criticism of bank bailouts during the financial crisis of 2007-2008, many governments pledged to end taxpayer subsidies to banks. If one accepts that actuarially fair deposit insurance is unrealistic, then the elimination of taxpayer subsidies to banks corresponds to revenue neutrality.

A major finding of this paper is that in the setting just described deposit insurance affects asset prices, relative to the prices that would otherwise prevail, if and only if the premia are not revenue neutral, but rather are favorable to banks in the aggregate. If premia are revenue neutral, so that the total subsidy to unsuccessful banks equals the total penalty to successful banks, there is no distortion in asset prices. This is true despite the fact that revenue neutrality is consistent with deposit insurance not being actuarially fair bank by bank and date by date, as just discussed. This conclusion is true in our model under both deposit-based and risky-asset-based insurance premia. Again, we believe that our result that revenue-neutral but actuarially unfair deposit insurance does not distort asset prices is novel.

There exists an important difference between the revenue-neutral equilibria depending on whether premia are charged against deposits or risky assets. In the former case there exists a continuum of equilibria that are revenue neutral, and not trivially so (that is, in which insurance collections and payouts are equal, and are also nonzero), whereas in the latter case there exists a continuum of equilibria all of which are trivially revenue neutral. The reason why there exist nontrivially revenue-neutral equilibria in one case but not the other is that under deposit-based premia the insurance payment is determined at date 0, when agents decide whether to direct their assets to commercial banks or shadow banks. This specification implies that at date 1 the insurance premium is a sunk cost: banks with high returns have no choice but to subsidize those with low returns through the deposit insurance program. In contrast, when premia are based on holdings of risky assets, premia are not a sunk cost:

commercial banks can avoid the insurance premium by selling risky assets to shadow banks. At a high enough premium rate, all commercial banks sell risky assets at date 1, leading to a situation in which deposit insurance is revenue neutral. In that case, at date 0 an agent would be willing to form a commercial bank only if it specialized in holding riskless assets at date 1. Being riskless, such banks never default and never benefit from an insurance transfer. The equilibrium is revenue neutral, but trivially so. There does not exist a nontrivial revenue-neutral equilibrium.

Analysis of bailouts of financial institutions deals with issues similar to those involved with deposit insurance. The principal difference is that, as the name implies, deposit insurance applies only to deposits, not to non-deposit liabilities such as bonds, or to equity. In contrast, the goal of bailouts is to avoid bankruptcy. In practice this implies that bank obligations to most creditors are guaranteed, not just deposits. Also, in bailouts enough support is supplied to assure that bank equity continues to maintain considerable value. With the appropriate modifications, the model to be presented can be used to analyze "too big to fail" and other topics related to bailouts.

The setting here is stylized, so as to focus on the essentials of the problem. As discussed in the conclusion, we expect that the qualitative results should carry over to settings in which the various restrictions that we impose are relaxed.

1   No Deposit Insurance

In this section a simple model without deposit insurance is specified. This model—in particular, the equilibrium prices of risky and riskless assets—will serve as a benchmark against which to compare the equilibrium prices of assets when commercial banks' deposits are insured. The equilibrium described in this section may be interpreted as that occurring under shadow banking, meaning that all assets are held by financial institutions that do not participate in the deposit insurance program.

There are three dates: 0, 1 and 2. Each member of a continuum of agents

has a date-0 endowment consisting of one unit each of a riskless asset and a risky asset. The riskless asset is costlessly storable from date 0 to date 1, and from date 1 to date 2, when it transforms into one unit of a consumption good. At date 1 each agent's holding of the risky asset is subject to a multiplicative productivity shock ε_1, different for different agents, so that it becomes ε_1 units of the asset at date 1. At date 2 another multiplicative shock ε_2 occurs, resulting in $\varepsilon_1\varepsilon_2$ units of the consumption good at date 2. The shocks ε_1 and ε_2 of each agent are uniformly distributed on the interval $[\underline{\varepsilon}, \overline{\varepsilon}]$, independently of each other and of the shocks of other agents. Agents consume at date 2 and are risk neutral.

Throughout this paper we identify the shocks with the holder of the risky asset rather than with the asset itself. For example, if one bank sells the risky asset to another at date 1, as will occur under deposit insurance, the newly acquired asset undergoes the date 2 shock of the buying bank, not that of the selling bank. Without this specification the assumption that asset returns are contemporaneously independent would imply that returns on diversified bank portfolios are deterministic with probability one, which would trivialize the analysis of deposit insurance.

It is assumed that each agent who elects to use banks allocates his endowment of risky and riskless assets to his own personal bank. He receives in exchange deposits d and bank equity e_0; the numeraire is date-2 consumption. Agents who, contrary to this assumption, created deposits in banks other than those they own would run the risk that the equity holders in those banks would transfer risk to them after the deposit terms are set. This agency problem, which has been widely discussed in the finance literature (see Kose John, Teresa A. John and Lemma W. Senbet (1991) for an application to deposit insurance) is the same agency problem that we study below, except that here the insurer takes the place of depositors as the bearer of the agency distortion. In the setting assumed here depositors avoid the agency problem by creating separate banks, inasmuch as agents in their role as equity holders have no motive to exploit themselves in their role as depositors. Thus assuming that each agent operates his own bank is an anticipation of the equilibrium more

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than a separate assumption.

The date-0 value of each bank's assets is $p_0 + q_0$, where p_0 is the unit price of the risky asset and q_0 is the unit price of the riskless asset, both to be determined as part of the equilibrium. The date-0 balance sheet identity for banks is $p_0 + q_0 = d + e_0$. The Miller-Modigliani theorem implies that in the absence of deposit insurance agents will be indifferent to the breakdown of $p_0 + q_0$ into d and e_0, so we can take the breakdown as exogenous.

Depositors are paid a gross return R at date 2 if ε_1 and ε_2 are such that the assets of the bank are worth at least Rd. Equity holders receive $\varepsilon_1 \varepsilon_2 + 1 - Rd$. If the value of bank assets is insufficient to pay off depositors fully, then all the assets are paid out to the depositors. In this representative agent environment no one has motivation to trade assets at any date, so in equilibrium each agent is willing to hold his endowment of assets in a bank until date 2. Banks are liquidated at date 2 and each agent consumes his own endowment of assets. The unit price of the risky asset at date 0 that supports the equilibrium is $p_0 = [\underline{\varepsilon} + \overline{\varepsilon}]^2/4$, equal to the expectation of $\varepsilon_1 \varepsilon_2$. The equilibrium date-0 price of the riskless asset is $q_0 = 1$. The corresponding date-1 unit price of the risky asset is $p_1 = E(\varepsilon_2) = [\underline{\varepsilon} + \overline{\varepsilon}]/2$. The equilibrium interest rate on deposits from date 0 to date 2 is the value of R that satisfies $d = E[\min(Rd, \varepsilon_1 \varepsilon_2 + 1)]$. Equity holders receive $\varepsilon_1 \varepsilon_2 + 1 - \min(Rd, \varepsilon_1 \varepsilon_2 + 1) = \max(\varepsilon_1 \varepsilon_2 + 1 - Rd, 0)$. Thus the expected gross return on equity is 1, and the same is true for deposits.

We utilize a version of the law of large numbers to characterize the sample distribution of the shocks, implying that the realizations of ε_1 and ε_2 are uniformly distributed, like the population distribution of each bank's ε_1 and ε_2.[6]

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[6]In appealing to "a version of the law of large numbers" we follow a standard practice in applied general equilibrium analysis of sidestepping without discussion a difficulty involved in modeling sample outcomes generated by a continuum of realizations of independent random variables: the difficulty is that for any $Y \in [\underline{\varepsilon}, \overline{\varepsilon}]$ the set of realizations for the event $\varepsilon_1 \leq Y$ is nonmeasurable with probability 1, implying that the usual characterization of the cumulative distribution function $P(y) = prob(Y \leq y)$ is not available. Thus the simplest justifications for the law of large numbers do not apply. This problem was first pointed out by Judd (1985) and Feldman and Gilles (1985). Several methods can be used to justify the law of large numbers; see Uhlig (1996). Most simply and intuitively, in this setting the sample distribution of a finite number of independent draws converges to the uniform

The bank balance sheets at dates 0 and 2 are shown as Tables 1 and 2.

2 Informal Description: Lump-sum Premia

We now modify the model by assuming that the bank insurer insures all commercial bank deposits. It does this by supplying funds at date 2 in an amount just sufficient to enable commercial banks with low realizations of ε_1 and ε_2 to pay depositors fully. The bank insurer makes no payment to equity holders of insolvent commercial banks, who therefore experience a 100 per cent loss. Instead of investing at commercial banks, agents can turn over their assets to shadow banks in exchange for deposits — these deposits are not insured. The shadow banks operate as outlined in the preceding section.

It is easiest to begin by assuming that deposit insurance is financed via a lump-sum tax t; doing so makes it possible to separate the effects of deposit insurance from the effects of how the insurance is financed. The tax is levied against the date-2 consumption of all agents. It is assumed throughout that consumption sets are unbounded below as well as above, ensuring that agents will always be able to pay the tax.

The tax is applied whether or not the agents choose to invest at commercial banks or shadow banks, whether or not the banks hold risky assets, and whether or not the banks turn out to be insolvent. For individual agents, investment decisions do not affect the amount of the tax they pay (although the choices all agents make collectively determine t). Thus t plays no role in determining agents' choices of whether to invest at a commercial bank or a shadow bank, what level of deposits to issue, or the asset transactions at date 1. The insurer sets the tax so that the revenue it generates is just sufficient to fund insurance payments to failed commercial banks. Total insurance payments are deterministic in the aggregate, there being no systematic risk in the economy.

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distribution as the number of draws becomes large. Therefore the appeal to the law of large numbers can be justified informally using a limiting argument. Duffie and Sun (2007, 2012) followed Feldman and Gilles in applying nonstandard analysis to demonstrate the limiting result rigorously.

At date 0 each agent chooses whether to allocate his endowments of risky and riskless assets to a commercial bank or to a shadow bank. We assume that each agent allocates equal amounts of risky and riskless assets to the commercial bank or the shadow bank, depending on his choice. In each case he designates part of the value of the transfer as deposits and the remainder as equity. The deposit insurer guarantees that the deposit component of the transfer to commercial banks yields a gross return of 1. The insurer makes no payment to depositors at shadow banks regardless of investment outcomes.

At date 1 holders of the risky asset (including both commercial and shadow banks) realize productivity shocks ε_1 which, as in the preceding section, are generally different for each bank. These shocks induce different equity holders at commercial banks to value the prospective transfer from the deposit insurer differently. Existence of this heterogeneity motivates commercial banks to trade assets among themselves and with shadow banks at date 1 after the realization of the shocks. Agents who operate shadow banks can participate in this trade even though they are not eligible for transfers from the deposit insurer at date 2. At date 2 the second productivity shock ε_2 is realized and banks are liquidated: deposits and equities are paid out and consumption takes place.

Commercial banks that are unable at date 2 to pay their depositors from their own assets receive a transfer from the insurance fund. The total amount paid out, and therefore also the total tax collected t, is nonrandom by virtue of "a version of the law of large numbers", as discussed above. Agents in their roles as both depositors and equity holders act to maximize expected date-2 consumption, which equals the sum of deposits and expected equity, net of the lump-sum tax. Expected date-2 consumption also equals the sum of the date-0 value of the endowments of the risky and riskless assets, net of the lump-sum tax.

Commercial bank balance sheets are shown in Tables 3, 4 and 5.

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3 Outline of the Model

We begin with a partial equilibrium analysis in which we assume that all agents set up commercial banks and choose the same level d of deposits at date 0. Initially we take d as given. In the last paragraph of this section we will broaden the analysis to determine equilibrium by considering whether agents are motivated to deviate from the assumed equilibrium by choosing a different level of d as deposits at commercial banks, or turning over their assets to shadow banks rather than commercial banks. The description in this section is informal; details are presented in Section 5.

We assume that both commercial and shadow banks cannot issue, retire or transfer deposits at date 1, implying that banks cannot use deposits to pay for asset purchases. Instead, banks draw down or augment their holdings of the riskless asset according to whether they are buyers or sellers of the risky asset. This device, although artificial, serves to ensure that asset transactions lie in a bounded interval, implying existence of optima. At date 2 risky assets are subject to random productivity shocks ε_2. Following these shocks deposits and bank equity are withdrawn in the form of the physical good, which the agents then consume.

To begin the characterization of the equilibrium for different values of deposits d at commercial banks, consider first the case in which d is so low that all commercial banks will be able to pay off their depositors regardless of their draws of ε_1 and ε_2, and regardless of how many units of the risky asset they buy or sell at date 1. The worst possible outcome for a commercial bank is to experience the productivity shock $\underline{\varepsilon}$ at date 1, exchange its endowment of the riskless asset for the risky asset at the equilibrium relative price π_1 and experience another draw of $\underline{\varepsilon}$ at date 2. If d is low enough that this bank is able to honor its obligation to depositors, then all commercial banks can do so. We denote the no-default region as Region I.

In Region I commercial banks with different realizations of ε_1 will not have comparative advantages or disadvantages in holding the risky asset, so we

can suppress trade in assets at date 1.[7] All commercial banks will pay off depositors at date 2 and pay whatever remains to equity holders. With no commercial bank failures, the deposit insurer will set t equal to zero. The equilibrium values of the risky asset are $[\underline{\varepsilon}+\bar{\varepsilon}]^2/4 = E(\varepsilon_1\varepsilon_2)$ at date 0 and $\pi_1 \equiv E(\varepsilon_1) = [\underline{\varepsilon} + \bar{\varepsilon}]/2$ at date 1, as when there is no insurance. The riskless asset has value 1 at dates 0 and 1. We label the maximum value of d consistent with all commercial banks honoring deposit obligations without an insurance transfer as d^{I-II}. We have

$$d \leq d^{I-II} \equiv (\underline{\varepsilon} + 1/\pi_1)\underline{\varepsilon} = (\underline{\varepsilon} + 2/(\underline{\varepsilon} + \bar{\varepsilon}))\underline{\varepsilon}$$

in Region I.

Now suppose that d is higher than d^{I-II}, so that commercial banks with low ε_1 will fail if ε_2 is also low. Commercial banks that know they will not fail (those with high realizations of ε_1) value risky and riskless assets as in Region I. If d is only slightly higher than d^{I-II} the large majority of commercial banks are in this group, and they dominate the few commercial banks with very low ε_1 that may fail. Therefore the equilibrium relative price of the risky asset that prevailed in Region I, $\pi_1 = (\underline{\varepsilon} + \bar{\varepsilon})/2$, will carry over to the present case, which we denote Region II.

Define $\widehat{\varepsilon}$ as the highest value of ε_1 such that commercial banks with $\varepsilon_1 < \widehat{\varepsilon}$ face the possibility of failure at date 2, assuming (as will be the case in equilibrium) that they buy the risky asset at date 1. Commercial banks with $\varepsilon_1 > \widehat{\varepsilon}$ will be indifferent as to whether or not to trade the risky asset, as just noted. Because the equilibrium relative price of risky and riskless assets reflects the valuation of commercial banks that will not fail, commercial banks that do face the possibility of failure will strictly prefer to buy as much of the risky asset as possible at the equilibrium price. Thus commercial banks with

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[7]As stated, there exists an element of circularity here, since the date-1 price π_1 was involved in the definition of Region I. The resolution lies in the fact that agents will be willing not to trade at date 1 only if π_1 is equal to the ratio of the expected payoffs of the risky and the riskless asset. Therefore we can take this price as that incorporated in the definition of Region I.

$\varepsilon_1 < \widehat{\varepsilon}$ will exchange all of their holdings of the riskless asset in favor of the risky asset. Commercial banks with $\varepsilon_1 > \widehat{\varepsilon}$ are net sellers of the risky asset in the aggregate, but are not necessarily at boundary points.

Define $p_1(\varepsilon_1)$ as the shadow price of the risky asset at date 1 for a commercial bank with date-1 shock ε_1, measured in units of date-2 consumption. Similarly, define $q_1(\varepsilon_1)$ as the date-1 shadow price of the riskless asset. We have $p_1(\varepsilon_1)/q_1(\varepsilon_1) = \pi_1$ for all ε_1, due to the fact that commercial banks can trade the risky asset for the riskless asset, or vice versa, at price π_1. In Region II, as noted, π_1 equals the relative price that would obtain in the absence of deposit insurance. For commercial banks with low values of ε_1 the shadow price of the risky asset strictly exceeds the value justified by its direct payoff (meaning its payoff excluding the component representing the present value of a transfer from the insurer in the event of failure). This is so because an increase in a commercial bank's holding of the risky asset increases the expected value of the transfer from the insurance fund, and this increase is reflected in its shadow price. Similarly, for commercial banks with low ε_1 the shadow price of the riskless asset exceeds that justified by its direct payoff, reflecting the fact that a commercial bank with low ε_1 does strictly better to exchange the riskless asset for the risky asset than to hold the riskless asset to maturity.

Commercial banks with high ε_1 will not fail, and therefore cannot benefit from a transfer from the insurer. For these banks the shadow price of the risky asset is π_1, which equals the proceeds from selling it at date 1. The shadow price of the riskless asset is 1.

We turn now from date-1 shadow prices of assets held by commercial banks to those at date 0, still assuming that d lies in Region II. The assumption that agents use equal amounts of risky and riskless assets to create banks implies that at date 0 risky and riskless assets are a composite commodity.[8] As such,

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[8]If we were to allow agents to create banks with unequal holdings of risky and riskless assets at date 0 we could define the date-0 shadow prices of risky and riskless assets separately. However, doing so would complicate the analysis. We assume below that the bank regulator puts an upper bound on commercial bank deposits, equal to the maximum amount such that all banks have nonzero probability at date 1 of remaining solvent at date 2. The calculation of this upper bound would be more involved under that generalization than under the present specification.

they do not have well-defined individual shadow prices at date 0. We are led to define a new composite commodity that consists of one unit of the risky asset and one unit of the riskless asset. The shadow price (again, relative to date-2 consumption) at date 0 of this composite commodity, which we denote k_0, is well defined. It equals the expectation of the product of ε_1 and the date-1 shadow price of the risky asset, a function of ε_1, plus the expectation of the date-1 shadow price of the riskless asset, also a function of ε_1. Since in Region II both date-1 shadow prices of assets equal or exceed the expectation of their date-2 direct payoffs (depending on ε_1), the date-0 shadow price of the composite commodity strictly exceeds the sum of their expected date-2 direct payoffs.

Higher values of d result in higher $\widehat{\varepsilon}$, implying that more commercial banks run the risk of failure. Thus more commercial banks strictly prefer to buy the risky asset at its equilibrium price, and fewer commercial banks are indifferent as to whether to sell or not. For d above a borderline value, which we label d^{II-III}, commercial banks buying the risky asset demand more of the asset than is available at the price $(\underline{\varepsilon} + \overline{\varepsilon})/2$ from the selling banks. In that case the date-1 equilibrium relative price of the risky asset must increase to a level above $(\underline{\varepsilon} + \overline{\varepsilon})/2$. We label such values of d as Region III.

At the higher relative price of the risky asset commercial banks with high ε_1 now strictly prefer to sell all their holdings of the risky asset rather than being indifferent as to whether or not to sell, as in Region II. Commercial banks buying the risky asset each purchase less of the asset at the equilibrium price than they would at the lower price $(\underline{\varepsilon} + \overline{\varepsilon})/2$ that prevails in Region II. As will be made clear in Section 5, for each d there exists a single pair $(\pi_1, \widehat{\varepsilon})$ that is consistent with market clearing.

In Region III, as in Region II, commercial banks with low values of ε_1 impute shadow values to both assets strictly higher than their direct date-2 payoffs justify. The lower any commercial bank's realization of ε_1 is, the higher is the value imputed to both assets (although, of course, the ratio of these valuations is π_1 for all commercial banks, since they trade at that price). Commercial banks with $\varepsilon_1 > \widehat{\varepsilon}$ impute value $p_1(\varepsilon_1) = \pi_1$ to the risky asset

because they can sell additional units at that price. This valuation strictly exceeds the direct expected payoff on the risky asset, contrary to the case in Region II. They impute the same value, 1, to the riskless asset as its payoff justifies.

The upper bound of Region III is the maximum level of d such that even commercial banks with low realizations of ε_1 (who therefore buy the risky asset at date 1) can avoid failure at date 2 given a sufficiently high realization of ε_2. We denote this upper bound for Region III by d^*. For $d > d^*$ commercial banks with sufficiently low values of ε_1 would be certain to fail at date 2. Allowing this possibility would cause problems for the analysis. Commercial banks that are certain to fail have no stake in the returns on their investments. They would be indifferent as to whether to buy or sell the risky asset at date 1, or do nothing: the date-2 value of the equity of such banks is zero under any of these courses of action. This indeterminacy in commercial banks' investment decisions would result in an indeterminacy in equilibrium prices, since at date 0 all commercial banks run the risk of being certain at date 1 of failing at date 2. Rather than complicate the analysis of the model by incorporating this case we rule out indeterminacy by assuming that the insurer enforces the restriction $d \leq d^*$, so that all commercial banks are constrained to choose levels of d such that for all realizations of ε_1 at date 1 they have positive probability of remaining solvent at date 2 . This restriction is empirically realistic: bank insurers, at least in theory, shut down commercial banks when failure is a foregone conclusion.[9]

The final step of the analysis consists of replacing the assumption that d is specified to equal some arbitrary number in the interval $[0, d^*]$ with the specification that agents set d to maximize expected date-2 consumption, equal

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[9]It would seem that one could avoid the indeterminacy problem by specifying that commercial banks that are certain to fail cannot trade assets. This restriction in fact does not resolve the problem. Even if commercial banks that are certain to fail are prohibited from trading assets, the imputed date-1 values of the risky asset for these banks are not well defined, implying that the equilibrium date-0 values of the risky and the riskless asset are indeterminate.

We choose to rule out these indeterminacies because they are consequences of the details of our model specification, and do not seem to correspond to real-world phenomena.

to the sum of deposits plus date-0 equity, less the lump-sum tax. Restricting attention to equilibria in which all agents choose the same d, which is without loss of generality in this section (but not so in general, as we will see below), the task is to find the value(s) of d such that an individual agent will not deviate from the value chosen by other agents. Maximizing expected utility involves maximizing the expected transfer from the deposit insurer, implying that in equilibrium agents direct their assets to commercial banks, where (unlike with shadow banks) they have some prospect of receiving a transfer from the insurer. Setting d equal to d^* is a dominant strategy against any value of d chosen by other agents. Therefore $d = d^*$ is an equilibrium in dominant strategies: commercial banks set deposits at the highest level permitted by the insurer.[10]

As anticipated in the discussion just presented, when deposit insurance is financed by lump-sum taxes agents never choose to turn over assets to shadow banks. Doing so implies that they would pay the lump-sum tax but would not benefit from insurance payments, which is dominated by depositing at commercial banks and setting $d = d^*$.

4   An Example

We now present an example of the equilibrium under deposit insurance. Our example will set $\bar{\varepsilon}$ to 2 and $\underline{\varepsilon}$ to 0.5, implying that the risky asset is equally likely to experience any outcome between a doubling of value and a halving at date 1, and similarly at date 2. Consider first the setting in which there is no deposit insurance, discussed in Section 1. In that case the value of the risky asset is $1.25^2 = 1.56$ at date 0 and 1.25 at date 1, regardless of the division of shadow-bank value into debt and equity. The riskless asset has price 1 at both dates. Thus the date-0 value of each agent's endowment is 2.56. If deposits d are set below $1 + \underline{\varepsilon}^2 = 1.25$ deposits are risk-free, implying that their return R equals 1. Figure 1 shows the equilibrium rate of return on deposits for higher

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[10]The observation that levered banks value deposit insurance more highly than unlevered banks, with the corollary that banks' motivation for adopting highly levered asset structures is precisely to maximize the value of deposit insurance, has been made in partial equilibrium settings before. See Keeley and Furlong (1990), for example.

values of d.

Now impose deposit insurance, so that R is guaranteed at 1 regardless of commercial bank solvency. The equilibrium level d^* of d equals 2.47. Figure plots k_0 and π_1 as functions of d for values of d between 0 and 2.47. The boundaries for the equilibrium regions discussed in Section 3 are as follows: $d^{I-II} = 0.65$ and $d^{II-III} = 1.16$.

The existence of deposit insurance increases the date-0 equilibrium price of the composite commodity, consisting of one unit of the risky and one unit the riskless asset, from 2.56 (calculated as, $1.56 + 1$) to 2.93. This increase in the value of endowments is exactly offset by the lump-sum tax t.

5   Formal Analysis

Here we discuss explicitly the determination of the equilibrium in Region III. The derivation of the equilibrium in the other regions is similar (although simpler), and does not need to be discussed separately.

A commercial bank with date-1 shock realization ε_1 must decide whether to buy or sell the risky asset. Let x be the net purchase of the risky asset, bounded by $-\varepsilon_1$ and $1/\pi_1$. Commercial banks choose x to maximize date-1 bank equity $e_1(\varepsilon_1)$, equal to the expected date-2 value of bank assets minus deposits. We have

$$e_1(\varepsilon_1) = \max_{x \in [-\varepsilon_1, 1/\pi_1]} E\{\max[(\varepsilon_1 + x)\varepsilon_2 + 1 - \pi_1 x - d, 0]|\varepsilon_1\} \tag{1}$$

and

$$x^*(\varepsilon_1) = \argmax_{x \in [-\varepsilon_1, 1/\pi_1]} e_1(\varepsilon_1). \tag{2}$$

Commercial bank equity at date 1 is convex in x because the term inside the mathematical expectation is a convex function of x, and expectation is a linear operator. The convexity of date 1 equity in x implies that we can take $x^*(\varepsilon_1)$ to be either $-\varepsilon_1$ or $1/\pi_1$. Thus $e_1(\varepsilon_1)$ equals $b(\varepsilon_1)$ if $x^*(\varepsilon_1) = 1/\pi_1$ (the bank buys the risky asset), or $s(\varepsilon_1)$ if $x^*(\varepsilon_1) = -\varepsilon_1$ (the bank sells the risky asset).

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The expressions for $b(\varepsilon_1)$ and $s(\varepsilon_1)$ are

$$b(\varepsilon_1) = \left(\frac{\overline{\varepsilon} - \widetilde{\varepsilon}(\varepsilon_1)}{\overline{\varepsilon} - \underline{\varepsilon}} \right) \left(\frac{\widetilde{\varepsilon}(\varepsilon_1) + \overline{\varepsilon}}{2} \right) (\varepsilon_1 + 1/\pi_1) - d \qquad (3)$$

and

$$s(\varepsilon_1) = 1 + \pi_1 \varepsilon_1 - d. \qquad (4)$$

In the expression for $b(\varepsilon_1)$ the first term on the right-hand side is the probability that the realization of ε_2 will be high enough for the bank to remain solvent, where $\widetilde{\varepsilon}(\varepsilon_1)$ denotes the value of ε_2 that just enables a commercial bank that buys the risky asset at date 1 to pay off its depositors (leaving nothing for the equity holders, but not requiring a transfer from the insurer). The second term is the expectation of ε_2 conditional on solvency (equal to the midpoint of the interval $[\widetilde{\varepsilon}(\varepsilon_1), \overline{\varepsilon}]$), and the third term is the bank's holding of the risky asset, including the assets just purchased. This expression reflects the fact that if ε_2 turns out to be lower than $\widetilde{\varepsilon}(\varepsilon_1)$ then commercial bank equity is zero, not the negative number that would obtain in the absence of deposit insurance if deposits had to be redeemed at par by the bank. The expression for $s(\varepsilon_1)$ reflects the fact that commercial banks that sell the risky asset will not fail at date 2, being risk-free.

For low values of ε_1 we have $b(\varepsilon_1) > s(\varepsilon_1)$, reflecting the fact that commercial banks with low ε_1 run a risk of failure if ε_2 is also low, in which case they benefit from a transfer from the insurer. Commercial banks with high ε_1 have lower risk of failing for any value of x, implying that they are less interested in offloading the downside of ε_2. For these banks we have $b(\varepsilon_1) < s(\varepsilon_1)$, reflecting the fact that π_1 exceeds the expected date-2 payoff of the risky asset. They elect to sell all their risky assets, resulting in a zero risk of failure.

The boundary $\widehat{\varepsilon}$ between the ε_1 realizations of commercial banks that buy the risky asset ($\varepsilon_1 < \widehat{\varepsilon}$) and those that sell it ($\varepsilon_1 > \widehat{\varepsilon}$) is the value of ε_1 such that the equity takes on the same value whether the bank buys or sells the risky asset:

$$b\left(\widehat{\varepsilon}\right) = s(\widehat{\varepsilon}). \qquad (5)$$

Equation (5), with equations (3) and (4) substituted in, gives an expression for $\widehat{\varepsilon}$ as a decreasing function of π_1. A second equation relating π_1 and $\widehat{\varepsilon}$ comes from the fact that the demand and supply of the risky asset can be written as functions of π_1 and ε_1. Setting the aggregate demand for the risky asset equal to aggregate supply yields

$$\frac{(\widehat{\varepsilon} - \underline{\varepsilon})}{\pi_1} = \frac{\overline{\varepsilon}^2 - \widehat{\varepsilon}^2}{2},\tag{6}$$

which defines $\widehat{\varepsilon}$ as an increasing function of π_1. We thus have two functions, (5) and (6), giving $\widehat{\varepsilon}$ as a function of π_1, one increasing and one decreasing. Equilibrium occurs at the intersection of these functions.

The expressions just derived for date-1 commercial bank equity, plus the bank balance sheet identities, allow computation of the shadow prices. For commercial banks selling the risky asset, total assets equal $(1 + \pi_1\varepsilon_1)q_1(\varepsilon_1)$, while bank deposits plus equity equals $1 + \pi_1\varepsilon_1$. It follows that $q_1(\varepsilon_1) = 1$, as noted in Section 3. For commercial banks buying the risky asset, the unit shadow value of their date-1 holding, $p_1(\varepsilon_1)$, is just equal to deposits d plus bank equity $b(\varepsilon_1)$, divided by the risky asset holding $\varepsilon_1 + 1/\pi_1$:

$$p_1(\varepsilon_1) = \frac{d + b(\varepsilon_1)}{\varepsilon_1 + 1/\pi_1},$$

from the date 1 bank balance sheet.

6   Deposit-Based Premia

Up to now it has been assumed that deposit insurance is financed using lump-sum taxes paid to the insurer by the agents. As noted, this specification has the convenient implication that the financing of deposit insurance can be ignored in determining the equilibrium. However, lump-sum financing of deposit insurance does not correspond to the real-world situation. In the United States banks have historically paid insurance premia to the Federal Deposit Insurance Corporation based either on the amount of deposits that

are insured or on the bank's assets. In this section we alter the model by assuming that the insurer charges commercial banks a sum equal to bank deposits multiplied by an insurance premium δ (for "deposits"). Commercial banks take δ as given.

In this section we use the term "revenue neutral" to mean that the revenue from the deposit-based insurance premium alone equals the total transfer to failed banks. We distinguish two types of revenue neutrality. Deposit insurance is nontrivially revenue neutral if the total revenue and total transfers are equal and nonzero. If the total revenue and total transfers are both zero, deposit insurance is trivially revenue neutral. If deposit insurance is not revenue neutral the shortfall or overage is made up in a lump-sum tax or transfer. The corresponding definitions apply in the following section. Therefore the environment assumed in Section 2 is a special case of those assumed in this and the following sections (insurance premia based on deposits or holdings of risky assets are zero).

It is easy to modify the model to allow for deposit-based insurance premia, although it turns out that the equilibria under deposit-based premia are different from those under lump-sum taxes. The details, being similar to those presented above, are deleted. The equilibria under deposit-based premia are generated by replacing (1) with

$$e_1(\varepsilon_1) = \max_{x \in [-\varepsilon_1, 1/\pi_1(\delta)]} E\{\max[(\varepsilon_1 + x)\varepsilon_2 + 1 - \pi_1(\delta)x - d(1 + \delta), 0]|\varepsilon_1\} \quad (7)$$

(this equation coincides with (1) if $\delta = 0$). As the equation above indicates, and as we will see in what follows, the relative price of the risky asset depends on δ.

Equation (7) is based on the specification that commercial banks with date-2 asset value net of deposits (equal to $(\varepsilon_1 + x)\varepsilon_2 + 1 - \pi_1(\delta)x - d$) exceeding δd pay δd to the insurer as a premium and distribute the remainder to equity holders. Commercial banks with asset value net of deposits less than δd fail. The insuring agency confiscates the assets of failed banks and pays the deposit liabilities of these banks in full. The insurance premium paid by failed banks

equals their asset value net of deposits if that number is positive, zero otherwise. Equity holders of failed banks experience a 100 percent loss. At date 2 depositors are paid their deposits, without interest, by all commercial banks regardless of returns on bank-held assets.

Agents can turn over their assets to shadow banks at date 0, as an alternative to allocating them to commercial banks. Shadow banks do not pay the insurance premium and do not receive a transfer from the insurer. Agents who start shadow banks do, however, pay the lump-sum tax. As we will see, and as one would expect, whether agents are willing to allocate their assets to shadow banks as an alternative to commercial banks depends on δ.

We will characterize equilibria for different values of δ. Suppose first that δ is set at a low level. Not surprisingly, the equilibrium in this case is qualitatively similar to that which obtains when the entire cost of deposit insurance is financed by a lump-sum tax, as analyzed above. All agents allocate their assets to commercial banks, since to do otherwise involves foregoing the possible payment from the deposit insurer. For the same reason, commercial banks will set deposits equal to the maximum level permitted by the insurer. As above, this limit equals the maximum deposit level such that the bank has a positive probability of surviving at date 2 for all values of the date-1 shock, assuming that the bank exchanges its holdings of the riskless asset for the risky asset at date 1 (recall that a bank must pay both deposits and the insurance premium to survive). This level, which depends on δ, is denoted $d^*(\delta)$. It is given by

$$d^*(\delta) = \frac{\overline{\varepsilon}(\underline{\varepsilon} + 1/\pi_1(\delta))}{1 + \delta}. \tag{8}$$

In the $\delta = 0$ case, both risky and riskless assets have equilibrium date-0 values that exceed expected direct payoffs. For low levels of δ most of the revenue required to finance the transfer to depositors at failed commercial banks is raised by lump-sum taxes.

Now consider higher values of δ. At the higher level of δ the expected payoff from starting commercial banks with $d = d^*(\delta)$ is decreased. At a certain value of δ, which we label $\underline{\delta}$, an agent's expected payoff from starting a shadow bank

equals his expected payoff from starting a commercial bank with $d = d^*(\delta)$. Accordingly, the aggregate value k_0 of agents' endowment is the same under a commercial bank as under a shadow bank, and agents are indifferent between allocating their endowment to either bank.

For $\delta = \underline{\delta}$ deposit insurance is revenue-favorable to commercial banks: the expected insurance payoff to owners of commercial banks setting d equal to $d^*(\underline{\delta})$ strictly exceeds the expected insurance premium paid by these banks. Thus starting a commercial bank strictly dominates starting a shadow bank and holding assets to maturity (and, by the same reasoning, dominates starting a commercial bank with $d = 0$ and holding assets to maturity). However, instead of holding assets to maturity, an agent starting a shadow bank can achieve the same expected payoff as an agent starting a commercial bank with $d = d^*(\underline{\delta})$ by selling his holding of the risky asset to commercial banks at date 1. Doing so attains a higher expected payoff than holding the risky asset to maturity because the risky asset trades at a premium at date 1. Commercial banks with $d = d^*(\underline{\delta})$ buy or sell the risky asset at date 1 according to the realization of ε_1, as outlined in Section 2. These banks pay the insurance premium according to the payment schedule described earlier, and benefit from a transfer from the insurer if they fail.

For $\delta = \underline{\delta}$ all agents form commercial banks with $d = d^*(\underline{\delta})$ (this, of course, is consistent with individual agents being indifferent as to whether to start a commercial bank or a shadow bank). When δ strictly exceeds $\underline{\delta}$ agents are still indifferent about whether to start a commercial bank or shadow bank. However, it is no longer true that all agents start commercial banks: a strictly positive measure of agents allocate assets to shadow banks at date 0, and then sell the risky asset to commercial banks at date 1, as outlined in the preceding paragraph. For each level of δ the measure of agents starting commercial banks with $d = d^*(\delta)$ is determined as part of the equilibrium as a decreasing (and strictly decreasing for $\delta > \underline{\delta}$) continuous function of δ. Thus when δ exceeds $\underline{\delta}$ by a small amount some agents will start shadow banks, but the majority will start commercial banks with $d = d^*(\delta)$. For higher values of δ a majority of agents will start shadow banks.

The result that for $\delta > \underline{\delta}$ some agents form shadow banks at date 0 and then sell off their holding of the risky asset at date 1 is unexpected. The reason for the date-1 sale of the risky asset is that the demand for risky assets comes from commercial banks with low realizations of ε_1. These banks are willing to pay a high price for the risky asset because they are likely to fail, and therefore to benefit from a transfer from the deposit insurer. At date 0 these agents do not yet know that they will be in danger of failing, so they would not pay a high price for the risky asset. Thus the transfer of the risky asset can only come at date 1, after the realizations of ε_1.

At higher levels of δ, $\pi_1(\delta)$ decreases toward the expected date-2 direct payoff of the risky asset, relative to the riskless asset. This fact implies that a sufficiently high value of δ, labeled $\overline{\delta}$, will result in an equilibrium in which the prices of risky and riskless assets equal (that is, no longer exceed) their respective expected direct payoffs. At $\delta = \overline{\delta}$ the deposit insurance program no longer favors either commercial or shadow banks relative to agents who simply hold assets to maturity: selling the risky asset at date 1 does not directly benefit shadow banks because for $\delta = \overline{\delta}$ deposit insurance does not distort asset prices in favor of risky assets. Starting commercial banks with $d = d^*(\overline{\delta})$ is not advantageous either because the expected gain from offloading the lower tail of the return distribution to the insurer is exactly offset by the expected insurance premium.

Deposit insurance is nontrivially revenue neutral at $\delta = \overline{\delta}$. We have seen that for $\delta < \overline{\delta}$ deposit insurance is revenue-favorable to banks (that is, it entails a subsidy financed by lump-sum taxes). For $\delta > \overline{\delta}$ deposit insurance is trivially revenue neutral: all agents avoid participating in the deposit insurance program by creating shadow banks or by creating commercial banks but not issuing deposits, so the insurer's revenue and expenditure both equal zero. We see that nontrivial revenue neutrality is a knife-edge case that divides revenue-nonneutrality ($\delta < \overline{\delta}$) from trivial revenue neutrality ($\delta > \overline{\delta}$).

Figures 3(a)-(d) and 4 calculate the equilibria for the numerical example with $\underline{\varepsilon} = 0.5$ and $\overline{\varepsilon} = 2$. The upper boundary value $\underline{\delta}$ for the region where all agents start commercial banks with $d = d^*(\delta)$ equals 0.10. As δ rises from

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0.10 to about 0.15 the proportion of agents who form commercial banks with $d = d^*(\delta)$ decreases from 100 per cent to about 60 per cent (Figure 3c). As the figure indicates, the proportion of commercial banks with $d = d^*(\delta)$ has a kink at about $\delta \simeq 0.15 \equiv \widetilde{\delta}$. The kink occurs because for δ less than $\widetilde{\delta}$ an increase in δ increases aggregate demand for the risky asset by increasing $\widehat{\varepsilon}$ (Figure 3a); recall that commercial banks with $\varepsilon_1 < \widehat{\varepsilon}$ are buyers of the risky asset, while those with $\varepsilon_1 \geq \widehat{\varepsilon}$ are sellers. For markets to clear, this increase in demand must be accompanied by an increase in the measure of shadow banks who supply the risky asset at date 1. At $\delta = \widetilde{\delta}$ all commercial banks are buyers of the risky asset at date 1, $\widehat{\varepsilon} = \overline{\varepsilon}$. Therefore the effect just described disappears for $\delta > \widetilde{\delta}$. Further increases in δ result in a much more gradual increase in the proportion of commercial banks buying the risky asset (there is still some effect because δ affects the equilibrium prices of risky and riskless assets, which in turn affect banks' date-1 trades in assets). Thus for values of δ greater than $\widetilde{\delta}$ about 60 per cent of agents start commercial banks with $d = d^*(\delta)$ and are buyers of the risky asset at date 1, while the remaining agents start shadow banks and are sellers of the risky asset.

The threshold $\overline{\delta}$ equals 0.21. Deposit insurance is non-trivially revenue neutral at that value of δ, and trivially so for higher values of δ. Figure 3d shows the insurer's expenditures and revenues as functions of δ. At $\delta = \overline{\delta}$ the mapping between δ and the measure of commercial banks that set d equal to $d^*(\overline{\delta})$ is multivalued: it ranges from zero to about 60 per cent. The multivalued mapping is a consequence of the fact that, at prevailing market prices, commercial banks are indifferent between setting d equal to zero or to $d = d^*(\overline{\delta})$, and shadow banks are indifferent between selling (or, for that matter, buying) the risky asset or not. Date 1 asset markets clear as long as aggregate demand of the risky asset from $d = d^*(\overline{\delta})$ banks is less than the net supply of the asset from shadow banks, as it is if fewer than 60 per cent of agents form commercial banks.

Figure 4 shows equilibrium date-0 valuations k_0 of the composite commodity on the balance sheets of commercial banks with $d = d^*(\delta)$ and of shadow banks.

7 Risky-Asset-Based Premia

Many analysts of banking recommend replacing deposit insurance premia based on deposit levels with premia based on asset risk: banks would be required to pay higher insurance premia to the extent that they hold more risky assets. In the United States the Federal Deposit Insurance Corporation is now moving in that direction. In this section we adapt the model so that insurance premia are based on holdings of the risky asset rather than deposits. We assume that commercial banks pay premia proportional to their after-trade date 1 holdings of the risky asset, with the factor of proportionality ρ (for "risky"). Here ρ is the same at all banks and does not depend on the magnitude of deposit liabilities. As with the deposit-based premia of the preceding section, any shortfalls or overages in the insurance fund are made up by a lump-sum tax or transfer.

The equilibria in this section are generated by modifying the expression for date-1 equity of commercial banks in the lump-sum case to include the risk-based insurance premium:

$$e_1(\varepsilon_1) = \max_{x \in [-\varepsilon_1, 1/\pi_1(\rho)]} \mathbb{E}\{[(\varepsilon_1 + x)(\varepsilon_2 - \rho) + 1 - \pi_1(\rho)x - d, 0]|\varepsilon_1\}. \quad (9)$$

Equation (9) shows that commercial banks pay the insurance premium $\rho(\varepsilon_1 + x)$ at date 2 provided their date-2 asset returns allow them to do so. It follows that commercial banks that sell their entire holding of the risky asset, $x = -\varepsilon_1$, do not pay an insurance premium. Commercial banks with assets net of deposit liabilities less than $\rho(\varepsilon_1 + x)$ fail. The insurance payment by failed banks equals the greater of their assets net of deposit liabilities and zero. The deposit insurer pays the depositors of failed banks in full, while enforcing a 100 per cent loss on equity holders.

Agents can also turn over their endowment to shadow banks. These banks do not pay the risk-based insurance premium and do not receive a transfer from the insurer. Agents who start shadow banks do pay the lump-sum tax. These specifications are the direct counterpart of the specification of deposit

insurance in the preceding section. As in the preceding section, we omit the details of the analysis.

We will characterize the equilibrium for various values of ρ. As with deposit-based premia of the preceding section, the maximum deposit level at commercial banks permitted by the insurer depends on ρ. This level is given by

$$d^*(\rho) = (\bar{\varepsilon} - \rho)(\underline{\varepsilon} + 1/\pi_1(\rho)). \tag{10}$$

The equilibrium when ρ equals zero reduces to the lump-sum case: all agents form commercial banks with $d = d^*(0)$ in order to take maximum advantage of the favorable deposit insurance program. At date 1 commercial banks with high values of ε_1 sell their holdings of the risky asset to commercial banks with low values of ε_1.

Suppose that ρ is set at a value slightly higher than zero. Doing so implies a higher cost of holding risky assets, thereby inducing some commercial banks with intermediate realizations of ε_1 who otherwise would be buyers to sell their holding of the risky asset at date 1. As a consequence, aggregate supply of the risky asset exceeds aggregate demand at the date-1 relative price that prevails for $\rho = 0$. It follows that, at the higher value of ρ, the date-1 relative price of the risky asset must decline for markets to clear. The date-0 expected payoff from starting a commercial bank with $d = d^*(\rho)$ decreases when ρ is increased from zero to the higher value.

As ρ is set at still higher values, $\pi_1(\rho)$ is lower, further decreasing the superiority of creating commercial banks and setting $d = d^*(\rho)$ over creating shadow banks at date 0. Eventually $\pi_1(\rho)$ drops below the relative price associated with the expected date-2 payoffs of risky versus riskless assets.[11] At a still higher value of ρ, which we label $\underline{\rho}$, the date-1 relative price of the risky asset is so far below its expected direct payoff that an individual agent's payoff from allocating his assets to a shadow bank at date 0 (thereby forego-

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[11]It is obvious that such a drop must come eventually, since at some level of ρ the premium cost of deposit insurance will exceed the expected benefit of the bankruptcy transfer. It is noteworthy, and somewhat counterintuitive, that this drop comes even under levels of ρ low enough that all banks choose to create commercial banks at date 0.

ing the possibility of a payout from the insurer, but also avoiding the deposit insurance premium) and buying the risky asset at date 1 is the same as his payoff from starting a commercial bank with $d = d^*(\rho)$. It follows that for $\rho = \underline{\rho}$ individual agents at date 0 are indifferent between creating commercial banks with $d = d^*(\underline{\rho})$ and creating shadow banks. For $\rho = \underline{\rho}$ all agents create commercial banks.

When ρ exceeds $\underline{\rho}$ by a small amount most agents create commercial banks with $d = d^*(\rho)$, but others create shadow banks and purchase $1/\pi_1(\rho)$ units of the risky asset at date 1 from commercial banks with high realizations of ε_1. The expected payoff from creating commercial banks equals that from creating shadow banks, implying that individual agents are indifferent about whether to turn over their assets to commercial banks or shadow banks. The proportion of agents who choose to form commercial banks and set d equal to $d^*(\rho)$ is continuous in the vicinity of $\underline{\rho}$ (and elsewhere), and is declining in ρ.

Commercial banks with $d = d^*(\rho)$ are buyers or sellers of the risky asset at date 1 depending on whether their realization of ε_1 is below or above $\widehat{\varepsilon}$. We have that $\pi_1(\rho)$ is high—but still lower than the present value of the payoff of the risky asset relative to the riskless asset—when ρ is high, reflecting the fact that high ρ induces many agents to create shadow banks, which are buyers of the risky asset at date 1. Unlike deposit-based premia of the preceding section, the date-1 relative price of the risky asset is thus a non-monotonic function of the risk-based insurance premium rate.

It is noteworthy that when insurance premia are based on holdings of risky assets (and satisfy $\underline{\rho} < \rho$), shadow banks are buyers of risky assets, rather than being sellers as when premia are based on deposits. This occurs because shadow banks are exempt from insurance premia, and occurs despite the fact that they will not receive a transfer in the event of failure, which by itself would confer a comparative advantage on commercial banks as holders of risky assets. Commercial banks with high realizations of ε_1 determine that the insurance premium exceeds the expected insurance transfer, so they sell the risky asset and avoid the insurance premium. For still higher values of ρ all commercial banks except those with very low realizations of ε_1 elect to sell the risky asset,

and they do so despite the fact that deposit insurance is revenue-favorable to commercial banks.

At a sufficiently high value of ρ, labeled $\bar{\rho}$, all commercial banks with $\varepsilon_1 > \underline{\varepsilon}$ elect to sell the risky asset. With all risky assets transferred to the shadow banks, the equilibrium date-1 relative price of the risky asset equals the ratio of the direct asset payoffs. At $\bar{\rho}$ the deposit insurance program is overpriced for all commercial banks excepting the banks with $\varepsilon_1 = \underline{\varepsilon}$ (a set of measure zero), who are indifferent about whether to participate in the deposit insurance program or not.

Risk-based deposit insurance is trivially revenue neutral at $\rho = \bar{\rho}$, and also for $\rho > \bar{\rho}$: being overpriced, deposit insurance results in all risky assets being held by shadow banks, who do not participate in deposit insurance.

Figures 5(a)-(d) and 6 present the equilibria for the numerical example with $\underline{\varepsilon} = 0.5$ and $\bar{\varepsilon} = 2$. In this example all agents create commercial banks with $d = d^*(\rho)$ as long as ρ is less than $\underline{\rho} = 0.307$. When $\rho = \underline{\rho}$, the date-1 relative price is around 1.15, which is below the relative expected direct payoff from one unit of the risky asset (see Figure 5b). When ρ exceeds $\underline{\rho}$, the measure of commercial banks with $d = d^*(\rho)$ declines (see Figure 5c). For example, when $\rho = 0.5$, only about 50 per cent of agents create commercial banks with $d = d^*(\rho)$. The date-1 relative price of the risky asset increases with ρ — when $\rho = 0.5$ the date-1 price is about 1.23 — reflecting the fact that the shadow banks buy the risky asset at date 1. The threshold $\bar{\rho}$ equals 0.74. At that value of ρ, the relative price of the risky asset equals 1.25, the relative expected direct payoff from one unit of the risky asset. As with deposit-based insurance premia, the mapping between the risk-based insurance premium rate ρ and the measure of commercial banks with $d = d^*(\rho)$ is multivalued at $\bar{\rho}$: it ranges from about 40 per cent to zero.

Comparing the results of this and the preceding section, it becomes clear that there are strong similarities between the equilibria that result from deposit-based premia and those resulting from risky-asset-based premia, but also one major difference. As for the similarities, in both cases low insurance rates involve a heavy subsidy to commercial banks, and agents respond to the resulting

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opportunity by allocating assets to commercial banks and setting deposit levels as high as possible. For insurance premia in the intermediate range—$\underline{\delta} < \delta < \overline{\delta}$ and $\underline{\rho} < \rho < \overline{\rho}$—some agents form commercial banks and others form shadow banks. Under both insurance regimes shadow banks benefit to the same extent as commercial banks from the subsidized deposit insurance. This is so for deposit-based premia because shadow banks sell the risky asset at prices that exceed its expected payoff, while for risky-asset-based premia shadow banks buy the risky asset at prices that are lower than expected payoffs.

The major difference between deposit-based premia and risky-asset-based premia is that in the former case there exists a nontrivially revenue-neutral equilibrium, while in the latter case there does not. Specifically, with $\delta = \overline{\delta}$, the deposit insurance program operates at a nonzero level, and results in undistorted asset prices. In contrast, with risky-asset-based premia, deposit insurance is revenue-favorable to commercial banks for $\rho < \overline{\rho}$, but commercial banking collapses for $\rho \geq \overline{\rho}$: all risky assets are held by shadow banks at date 1. To the extent that policy-makers want to implement an insurance program that minimizes asset price distortions (and to the extent that the present very simple model is thought to capture a feature of reality), this is an important finding.

The difference between equilibria with deposit-based and risky-asset-premia comes from the fact that, after setting the level of deposits at date 0, agents cannot avoid the insurance premium by reducing the level of deposits at date 1. As a consequence, commercial banks with high realizations of ε_1 pay the insurance premium even though they are unlikely to benefit from an insurance transfer. Under risky-asset-based premia commercial banks with high realizations of ε_1 are equally unlikely to benefit from an insurance transfer, but, in contrast to the case of deposit-based premia, they can avoid the insurance premium by selling the risky asset at date 1.

8 Conclusion

We have shown that deposit insurance financed by lump-sum taxes induces commercial banks to choose maximally levered—and therefore maximally risky—portfolios. This conclusion reflects a basic fact about deposit insurance: the expected net payoff on risky assets rises more than in proportion to the amount of risk held, predisposing banks to all-or-nothing solutions in terms of risk-bearing. We relied on risk neutrality to demonstrate this result, but it seems likely that the convexity property that underlies it carries over to more general settings.

If deposit insurance premia are based on deposit levels or risky asset holdings rather than lump-sum taxes, and if in addition premium levels are equal to or below (but not too much below) the revenue neutral level, then the date-0 market value of shadow banks equals that of commercial banks. This is so even though commercial banks receive payouts from the insurer, and these payouts strictly exceed the insurance premia (except in the special case of revenue neutrality, when they are equal). Shadow banks have the opportunity to sell risky assets at high prices at date 1 when insurance premia are based on deposits, or buy risky assets at low prices when insurance premia are based on risky asset holdings (again, except in the case of revenue neutrality, when asset prices are undistorted by deposit insurance). Thus except for extremely high or extremely low premium levels, commercial banks and shadow banks coexist in equilibrium, and individual agents are indifferent as to which type of bank to create. For extremely low values of the premium all wealth-holders direct their assets to commercial banks, while for extremely high levels they direct assets to shadow banks (or commercial banks that do not issue deposits or sell all their risky assets at date 1, depending on whether premia are based on deposits or risky asset holdings).

Thus our conclusion is that asset prices are distorted as a consequence of deposit insurance to the extent that the insurance program is financed using lump-sum taxes. This is a surprising conclusion inasmuch as lump-sum taxes are usually associated with the absence of distortion, not its presence. The

resolution of this anomaly lies in the fact that we are dealing with a general equilibrium model: in our setting lump-sum taxes are essentially the same thing (via the budget identities and market-clearing conditions) as the subsidy implied by underpriced deposit insurance. Accordingly, saying that lump-sum taxation causes the distortion is essentially the same thing as saying that underpriced deposit insurance causes the distortion. The latter formulation is not at all surprising.

A major goal of the Glass-Steagall Act was to separate commercial banking from investment banking, with commercial banks investing in safe assets only, and investment banks taking more risk. We saw in the preceding section that replacing deposit-based insurance premia with risky-asset-based premia has the desired effect: it encourages commercial banks to move toward lower-risk portfolios. In recent years, the Glass-Steagall Act has essentially been repealed. If one rejects the goal of insulating commercial banking from risk, then an institution like deposit insurance that results in risk being transferred from one class of institution to another serves no social purpose, at least to the extent that uninsured institutions are vulnerable to bank runs.

Presumably government agencies that insure bank deposits wish to minimize distortions induced by the insurance. In the present context this consideration favors implementation of revenue-neutral deposit insurance. However, we observed that, depending on the precise specification, there may or may not exist equilibria that are nontrivially revenue neutral. We are not certain whether the version of our model in which there exist nontrivial revenue-neutral equilibria is closer to the real world than the version in which there exists no such equilibrium. Obviously this is a matter that requires further investigation, particularly if proposals to base deposit insurance on holdings of risky assets, or to extend deposit insurance to institutions other than commercial banks, are implemented.

Our model incorporated several major simplifications in order to make the economic mechanisms clear:

- It was assumed that the riskiness of various bank investments can be unambiguously and accurately evaluated. This presumption is not remotely

accurate; consider the senior tranches of US mortgage-backed securities, which were treated by many investors, and also the rating agencies and some bank regulators, as virtually equivalent in riskiness to US Treasury bonds. The recent financial crisis, of course, showed otherwise.

- It was presumed that there exists a well-defined optimal bank response to deposit insurance; that is, that it is possible to determine exactly what banks would do in order to game the deposit insurance system to the extent implied by equity value maximization. Our model was structured to make this calculation possible, but in the real world it is hard to determine what meaning to attach to the idea of gaming the regulatory regime to the maximum extent feasible. In many theoretical settings deposit insurance would motivate commercial banks to hold an infinite long position in some assets and an infinite short position in others, which is inconsistent with existence of a well-defined equilibrium.

- We assumed that agents are risk neutral. Under universal risk-neutrality all feasible allocations are welfare equivalent, assuming, as we have, that deposit insurance that is not revenue-neutral is accompanied by the appropriate lump-sum taxes or transfers. If agents are risk averse this conclusion may fail due to the existence of welfare effects according to how deposit insurance is financed. It is clear that our major conclusion—that the presence of revenue-neutral deposit insurance of commercial bank deposits is consistent with existence of uninsured shadow banks under the asset prices that would prevail in the absence of deposit insurance— would carry over to settings characterized by low risk aversion. This is so because, by continuity, the preference of insured banks for corner optima under risk neutrality would carry over to settings with low degrees of risk aversion. Therefore the equilibrium would be essentially the same as under risk neutrality. However, for higher risk aversion the convexity property we derived might fail, invalidating our characterization of the equilibrium. However, it is not clear that this would necessarily occur: bank owners might still prefer to take maximal risk so as to benefit max-

imally from deposit insurance, with bank stockholders using assets other than bank equity (insured bank deposits, for example) to delever their portfolios and thereby mitigate risk exposure. If so, our conclusions may carry over at least qualitatively to the case of nontrivial risk aversion.

One might object to our specification on other grounds. There is no reason even to have deposit insurance in our environment. Further, banks themselves do not serve any social purpose in the setting that we hypothesize: as we noted, the equilibrium welfare of agents is the same with and without banks. This is a limitation of our model, but not necessarily a critical one. Economists have long postulated models populated by firms, where firms are characterized solely by production functions, with owners of any of the productive factors viewed as paying the others for services rendered. Doing so was recognized as a simplification, but one that does not necessarily invalidate conclusions drawn about the resulting equilibria. The specification here does not explicitly incorporate differences between commercial banks and shadow banks in, for example, their ability to monitor borrowers. Our conclusions reflect this specification, but this does not imply that they do not apply in more general settings.

The fact that there is no motivation for deposit insurance in our model invites comparison with Diamond and Dybvig (1983). The Diamond-Dybvig model is predicated on the presumption that banks play an essential role as financial intermediaries. However, Diamond and Dybvig provided no explanation of why there do not exist the securities markets that could bring about an allocation that, being efficient, would Pareto-dominate the allocation achieved under a banking system. In their setting agents have no incentive to misrepresent their type, so there is no reason to exclude the relevant financial markets. Given the existence of complete financial markets, however, banks would disappear, as shown by Freixas and Rochet (2008). Thus the absence in our setting of an explicit rationale for the existence of financial intermediaries, if it is a problem, has an exact parallel in the Diamond-Dybvig model (and, for that matter, most other banking models, which justify the existence of banks in a setting where there exist no other financial markets).

We believe that our conclusions do not depend critically on our simplifying assumptions, although the demonstrations of their validity would be more complex in less restricted settings. If so, the exercise reported in this paper provides a useful guide in thinking about real-world regulation of the banking system. Making the transition from toy models such as ours to reality involves replacing the stylized model of this paper with a more general model that connects more closely with the real world. Such a model could in principle be calibrated, leading to conclusions that can be taken seriously empirically. We have just enumerated reasons why providing such a model will be difficult, but we still think that attempting to do so is an appropriate direction for future work.

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Table 1: Shadow bank balance sheet at date 0.

Assets	Liabilities
p_0 (**risky asset**)	d (**deposits**)
q_0 (**riskless asset**)	$e_0 = p_0 + q_0 - d$ (**equity**)

Table 2: Shadow bank balance sheet at date 2, under deposit insurance financed entirely with lump-sum taxes.

Assets	Liabilities
$\varepsilon_1 \varepsilon_2$ (**risky asset**)	$\min(Rd, \varepsilon_1 \varepsilon_2 + 1)$ (**deposits**)
1 (**riskless asset**)	$\max(0, \varepsilon_1 \varepsilon_2 + 1 - Rd)$ (**equity**)

Table 3: Commercial bank balance sheet at date 0.

Assets	Liabilities
k_0 (**risky + riskless asset**)	d (**deposits**)
	$e_0 = p_0 + q_0 - d$ (**equity**)

Table 4: Commercial bank balance sheet at date 1 after portfolio adjustments. The function $p_1(\varepsilon_1)$ gives the date 1 unit value of the risky asset for a bank that gets the shock ε_1 and buys or sells x units of the risky asset; the function $q_1(\varepsilon_1)$ does the same for the riskless asset. The variable π_1 denotes the price of the risky asset, relative to the riskless asset.

Assets	Liabilities
$p_1(\varepsilon_1)(\varepsilon_1 + x)$ (**risky asset**)	d (**deposits**)
$q_1(\varepsilon_1)(1 - \pi_1 x)$ (**riskless asset**)	$e_1(\varepsilon_1) = (\varepsilon_1 + x)p_1(\varepsilon_1) + q_1(\varepsilon_1)(1 - \pi_1 x) - d$ (**equity**)

Table 5: Commercial bank balance sheet at date 2, under deposit insurance financed entirely with lump=sum taxes.

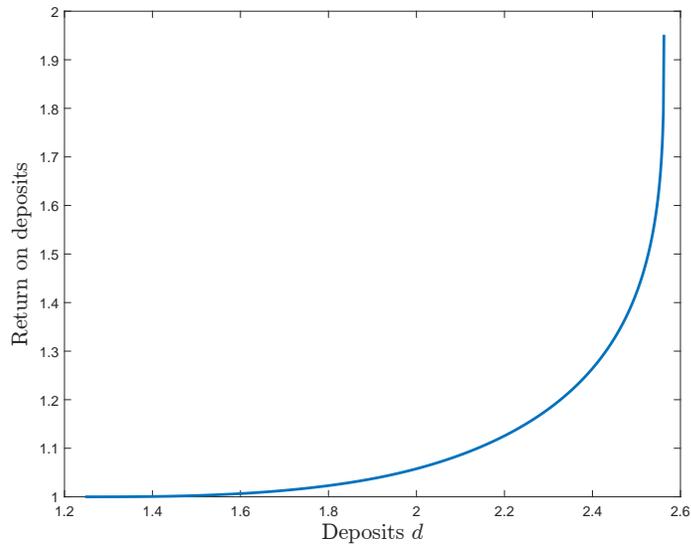| Assets	Liabilities
$(\varepsilon_1 + x)\varepsilon_2$ **(risky asset)**	d **(deposits)**
$1 - \pi_1 x$ **(riskless asset)**	
$\max[-\{(\varepsilon_1 + x)\varepsilon_2 + 1 - \pi_1 x - d\}, 0]$ **(insurance payment)**	$e_2 = \max[(\varepsilon_1 + x)\varepsilon_2 + 1 - \pi_1 x - d, 0]$ **(equity)**

Figure 1: Equilibrium rate of return on deposits in the absence of deposit insurance.
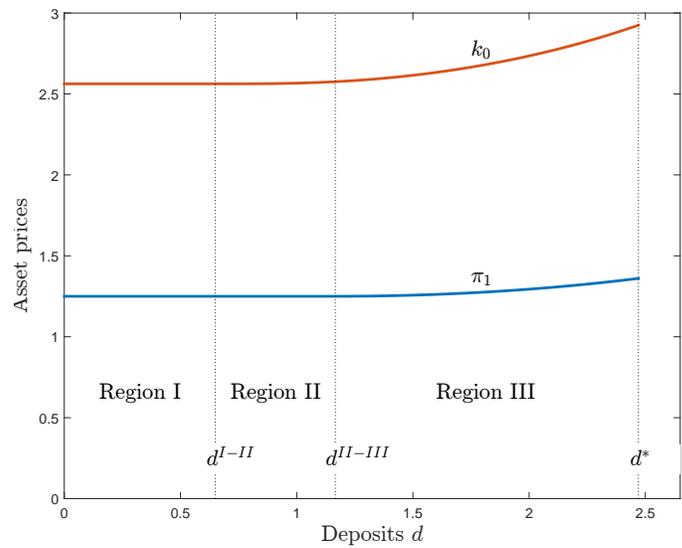
Figure 2: Asset prices for various levels of deposits when deposit insurance is financed using lump-sum taxes. In equilibrium, agents set d to its highest admissible value, $d = d^* = 2.47$.
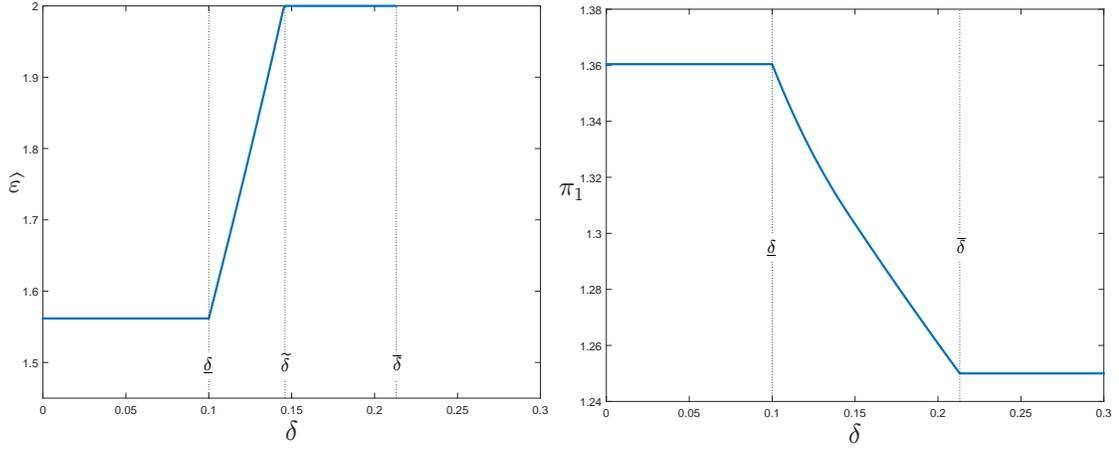
(a) The threshold $\widehat{\varepsilon}$ is that value of the date-1 shock ε_1 at which commercial banks with $d = d^*(\delta)$ switch from being buyers ($\varepsilon_1 < \widehat{\varepsilon}$) to sellers ($\varepsilon_1 \geq \widehat{\varepsilon}$) of the risky asset.

(b) Relative price of the risky asset at date 1. All agents start commercial banks with $d = d^*(\delta)$ when $\delta < \underline{\delta}$; shadow banks and $d = d^*(\delta)$ commercial banks coexist when $\underline{\delta} < \delta \leq \overline{\delta}$; all agents start shadow banks or nominal commercial banks ($d = 0$) when $\delta > \overline{\delta}$.
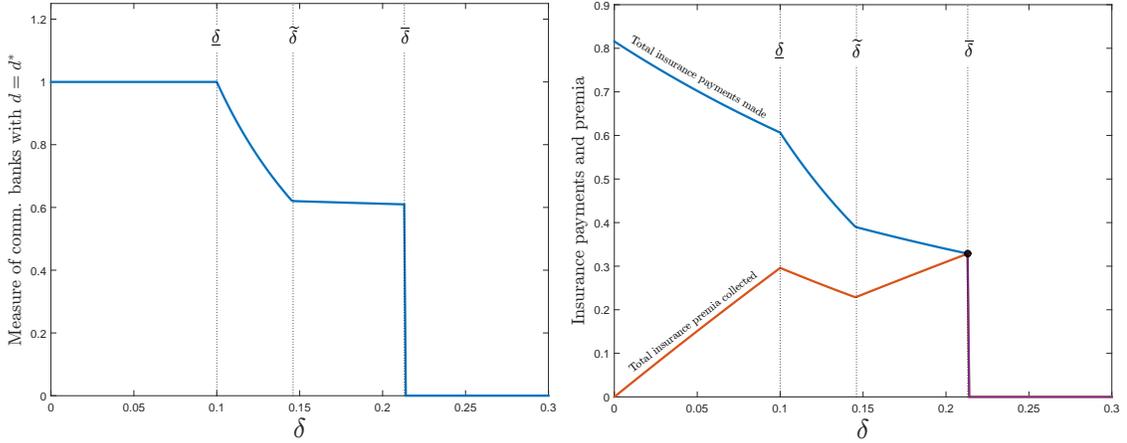
(c) The measure of agents that start commercial banks with $d = d^*(\delta)$.

(d) The total insurance premia collected and the total insurance payments made by the insuring agency. Deposit insurance is non-trivially revenue neutral when $\delta = \overline{\delta}$, and trivially so when $\delta > \overline{\delta}$.

Figure 3: Equilibrium under insurance premia based on deposit levels.
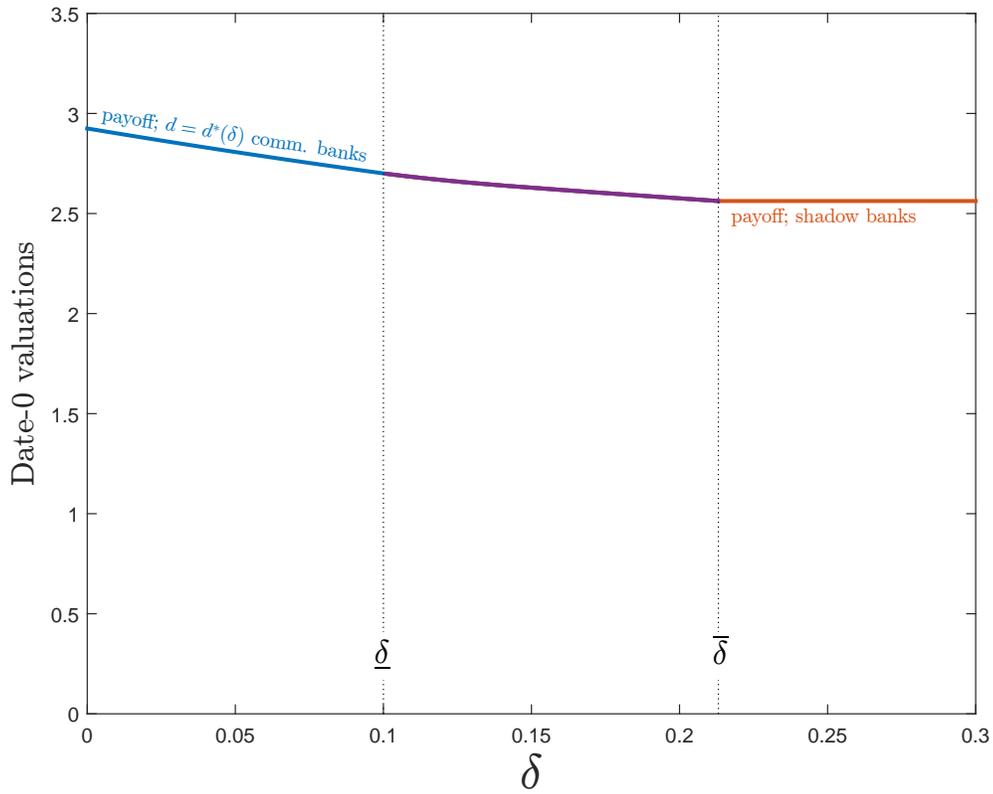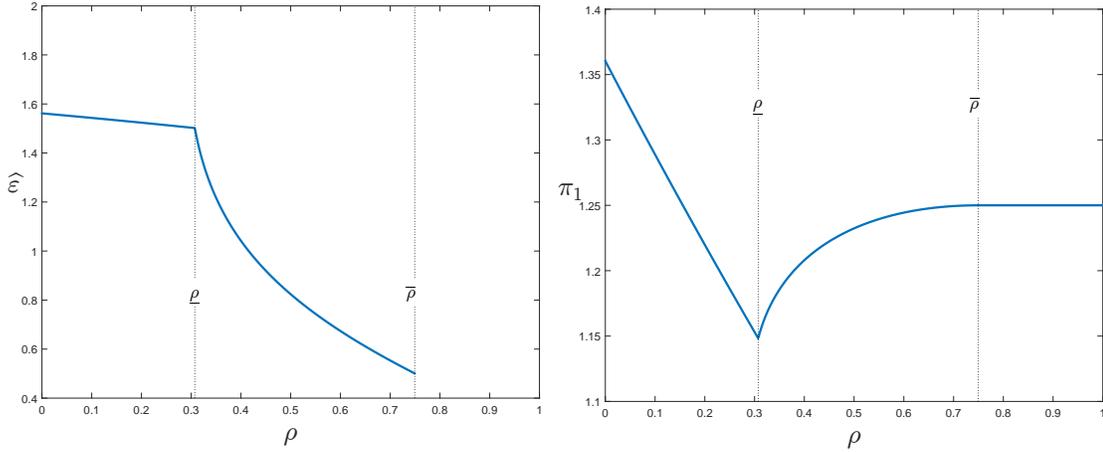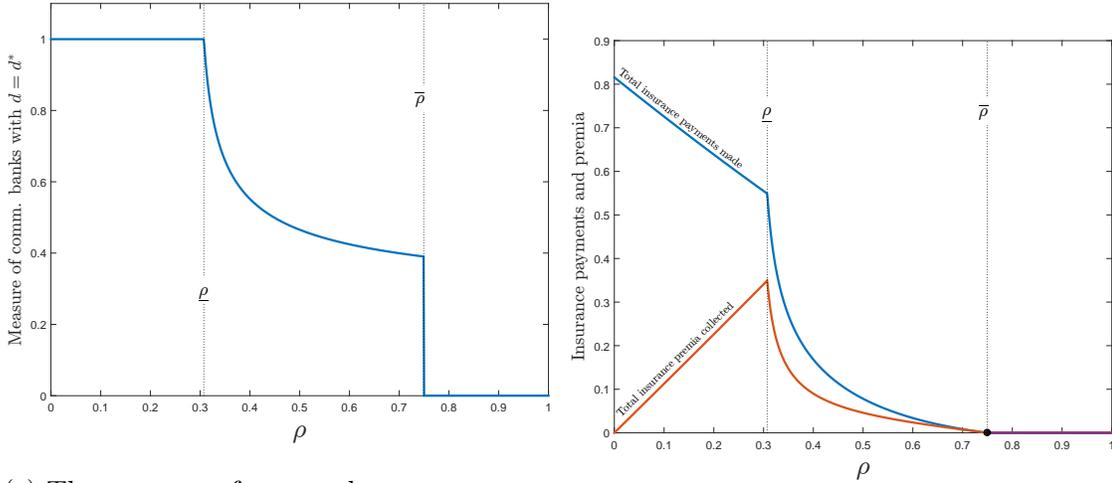
Figure 4: Date-0 valuation of the endowment by commercial banks with $d = d^*(\delta)$ (blue lines) and shadow banks (red lines), when insurance premia are based on deposit levels. For $\underline{\delta} < \delta < \overline{\delta}$ the line is purple because both commercial banks and shadow banks are being created. The difference between the value of the payoff line and 2.56 represents the aggregate lump-sum tax.

(a) The threshold $\widehat{\varepsilon}$ is that value of the date-1 shock ε_1 at which commercial banks with $d = d^*(\rho)$ switch from being buyers ($\varepsilon_1 < \widehat{\varepsilon}$) to sellers ($\varepsilon_1 \geq \widehat{\varepsilon}$) of the risky asset.

(b) Relative price of the risky asset at date 1. All agents start commercial banks with $d = d^*(\rho)$ when $\rho < \underline{\rho}$; shadow banks and $d = d^*(\rho)$ commercial banks coexist when $\underline{\rho} < \rho \leq \overline{\rho}$; all agents start shadow banks or commercial banks that sell their entire holding of the risky asset when $\rho > \overline{\rho}$.

(c) The measure of agents that start commercial banks with $d = d^*(\rho)$.

(d) The total insurance premia collected and the total insurance payments made by the insuring agency. Deposit insurance is trivially revenue neutral for all $\rho \geq \overline{\rho}$.

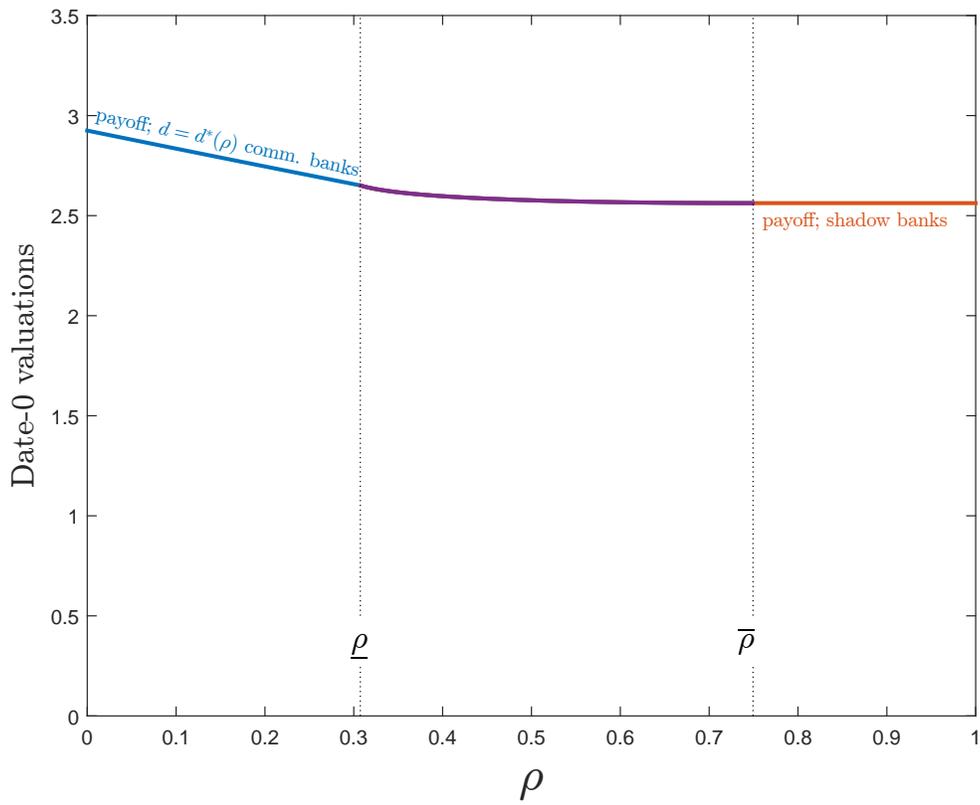Figure 5: Equilibrium under insurance premia based on risky-asset holdings.

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Figure 6: Date 0 valuation of the endowment by commercial banks with $d = d^*(\rho)$ and shadow banks, when insurance premia are based on holdings of the risky asset. Again, for $\underline{\rho} < \rho < \overline{\rho}$, the line is purple because both commercial banks and shadow banks are being created.